Utility-based Decisions

UMaine COS 470/570 – Introduction to AI

Spring 2019

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Utility-based reasoning

2.1

So far...

- We have explored reflex agents
- We have explored two types of goal-based agents:
  - Search agents
  - Planning agents
- What about finding the *best* solution to a goal?

2.2

Reflex-based utility agents

2.3

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- But: where to get $U(s)$ or $Q(a, s)$?

2.3

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2.4

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Sequential decision problems

Sequential decision problems

- Make sequence of action choices → goal state
- Planning is sequential decision problem
- But here:
  - Take (or find) sequence of actions → goal
  - Pick the *best* action in any state with respect to goal

3.2

Sequential decision problems

- What information can we use?
- Let $R(s)$ = reward for state *s*
- May be able to find R, since it's local
- Many states may have 0 reward:

$$s_0 \to a_1 \to s_1 \to a_2 \to \cdots a_n \to s_n$$
$$R(s_0) = R(s_1) = \cdots R(s_{n-1}) = 0$$

  - E.g., games, sometimes real world

3.3

Markov decision processes

- Formulate SDP as <S,A,T>:
  - *S* = states; distinguished state S_0
  - *A* = actions; $A(s)$ = all actions available in *s*
  - *T* = transition model
- *Markov decision process* (MDP):
  - *Fully-observable* environment (for now)
  - Transitions are Markovian
  - Stochastic action outcomes: $P(s'|s, a)$
  - Rewards additive over sequence of states (*environment history*)

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Policies

- What is solution to an MDP?
- Not just sequence of actions:
  - S_0 could be any *s*
  - Stochastic environment could ⇒ not reaching goal state
- Solution is a *policy* π:
  - $\pi(s)$ = action to take in state *s*
  - Agent always knows what to do next
  - Policy π ⇒ different environment histories (stochastic env.)
  - *Expected utility* of π
- *Optimal policy* π^* ⇒ highest expected utility
- π (or π^*):
  - is description of simple reflex agent
  - computed from info used by utility-based agent

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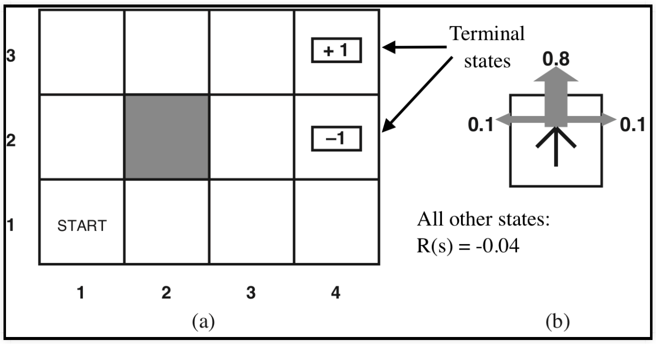
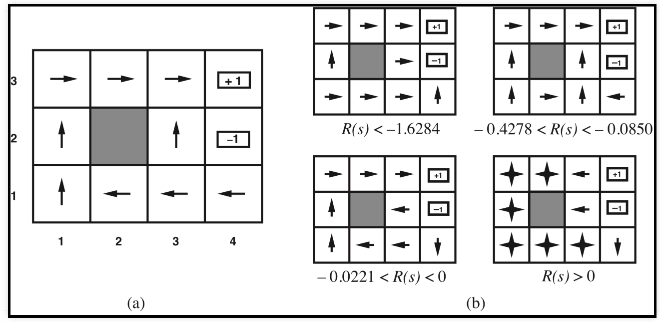
Example world



Some optimal policies



Utilities

- Reward $R(s)$: just depends on s
- *Utility* $U(s)$ of state depends on environment history h
$$U_h([s_0, s_1, s_2, \cdots]) = R(s_0) + \gamma R(s_1) + \gamma R(s_2) + \cdots$$
for *discount factor* $0 \le \gamma \le 1$
- Discount factor:
  - $\gamma < 1 \Rightarrow$ future rewards not as important as immediate ones
  - $\gamma = 1$: additive rewards

Utilities

- Finite or infinite *horizon*?
- Finite: game over after some time
  - Optimal policy: *nonstationary* with respect to different horizon
  - Short horizon: may choose shorter, but less optimal (or riskier) paths
  - Longer horizon: maybe more time to take longer, better paths
- Infinite: game *could* go on forever
  - Optimal policy is *stationary*
  - Optimal action depends only on state
  - Simpler to compute

Utilities

- Given a policy, can define utility of a state
$$U^\pi(s) = E[\sum_{t=0}^{\infty} \gamma^t R(S_t)]$$
where:
  - S_t is state reached at time t
  - Expected value $E(X) = \sum_i x_i P(X = x_i)$
  - Here, expectation is over prob. dist. of state sequences
- $\pi^* = \operatorname*{argmax}_\pi U^\pi(s)$
- True utility of s is $U^\pi(s) = U(s)$

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Optimal policy

- Kind of backward – what we want is π^*
- Can compute π^* if know $U(s)$ for all states
$$\pi^*(s) = \operatorname*{argmax}_{a \in A(s)} \sum_{s'} P(s' | s, a) U(s')$$
- But we said $U(s) = U^\pi(s)$ – which depends on π!
- How to compute?

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Bellman equation

- $U(s) = R(s) +$ expected discounted utility of next state
$$U(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' | s, a) U(s')$$
- This is the *Bellman equation*
- n states $\Rightarrow$ n Bellman equations (one per state)
- Also n unknowns – utilities for states
- Can we solve via linear algebra?
  - Problem: $\max$ is nonlinear
  - So no...

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Value iteration algorithm

- Can't directly solve the Bellman equations
- Instead:
  - Start with arbitrary values for $U(\cdot)$
  - For each s, do a *Bellman update*: calculate RHS → $U(s)$
  - Repeat until reach equilibrium (or change < some δ)
- Bellman update step:
$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' | s, a) U_i(s')$$

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Algorithm

function VALUE-ITERATION(mdp, ϵ) **returns** a utility function
inputs: mdp, an MDP with states S, actions $A(s)$, transition model $P(s' \mid s, a)$, rewards $R(s)$, discount γ
ϵ, the maximum error allowed in the utility of any state
local variables: U, U', vectors of utilities for states in S, initially zero
δ, the maximum change in the utility of any state in an iteration

repeat
$U \leftarrow U'; \delta \leftarrow 0$
for each state s **in** S **do**
$U'[s] \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' \mid s, a)\, U[s']$
if $|U'[s] - U[s]| > \delta$ **then** $\delta \leftarrow |U'[s] - U[s]|$
until $\delta < \epsilon(1-\gamma)/\gamma$
return U

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Example

3	0.812	0.868	0.918	+ 1
2	0.762		0.660	–1
1	0.705	0.655	0.611	0.388
	1	2	3	4

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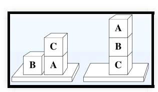
In-class exercise: POP

- No book/notes or online resources
- Show how POP would solve this problem (the Sussman anomaly):



  - Initial state: on(B,table), on(A, table), stacked(C,A)
  - Goal state: stacked(A,B), stacked(B,C)
  - Operators:
    - unstack(x,y) – take x off y (and arm will be holding it afteward)
    - stack(x,y) – put x (which the arm is holding) on y
    - pickup(x) – pick up x from the table
    - putdown(x) – put s (which the arm is holding) on the table

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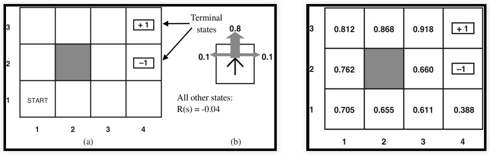
In-class exercise: MDPs

1. Given the example world:



- Use value iteration to find the utilities of the states – stop after 2 iterations
- How do your values compare with those gotten by R&N (above)?

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In-class exercise: MDPs

1. Draw a transition diagram for the Sussman anomaly
   - Use only the actions stack, unstack, putdown, pickup
   - Assume that with P(0.1), the arm drops the block when it's trying to stack it
   - Assume with P(0.2), the arm drops the block when it picks it up off the table or off another block

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POMDPs

- Assumed environment was fully-observable - but not always the case
- Environment *partially-observable* ⇒ not sure which state we're in!
  - Sensor uncertainty, sensor incompleteness, incomplete knowledge about interpretation
  - Hidden properties of world ("hidden variables")

$$\text{percept} \Rightarrow s_a | s_b | \cdots$$

- ⇒ *Partially-observable Markov decision process* (POMDP): much harder
- Real world *is* a POMDP

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POMDPs

- Action in POMDP ⇒ belief state
- Can reason over belief states
- In fact: POMDP ⇒ MDP of belief states
- Can do value iteration to find optimal policy for POMDPs

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Summary

- MDP: If we have a model of the environment and reward function, we can learn the optimal policy
- POMDP: Can still do it, using belief state MDP
- But what if we *don't* have an environment model or reward function?

$$\Rightarrow \quad \textit{reinforcement learning}$$

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