Constraint Satisfaction

UMaine COS 470/570 – Introduction to AI

Spring 2019

Search

Search so far. . .

- Uninformed search – nothing known about state space
- Heuristic search – *something* known, at least – defeasible
- Both: searching for a state with little internal structure
- Many problems: state has internal structure
- Important class of problems: state is assignment of *values* to *variables*

Examples

- Cryptarithmetic: Assign 0–9 uniquely to letters so that a symbolic expression is valid

```
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+MORE
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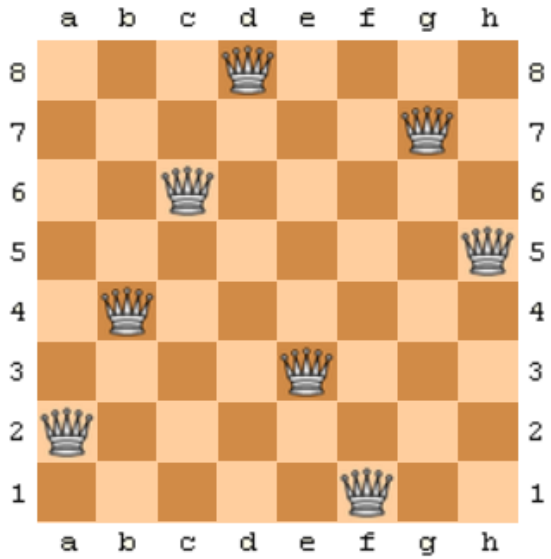
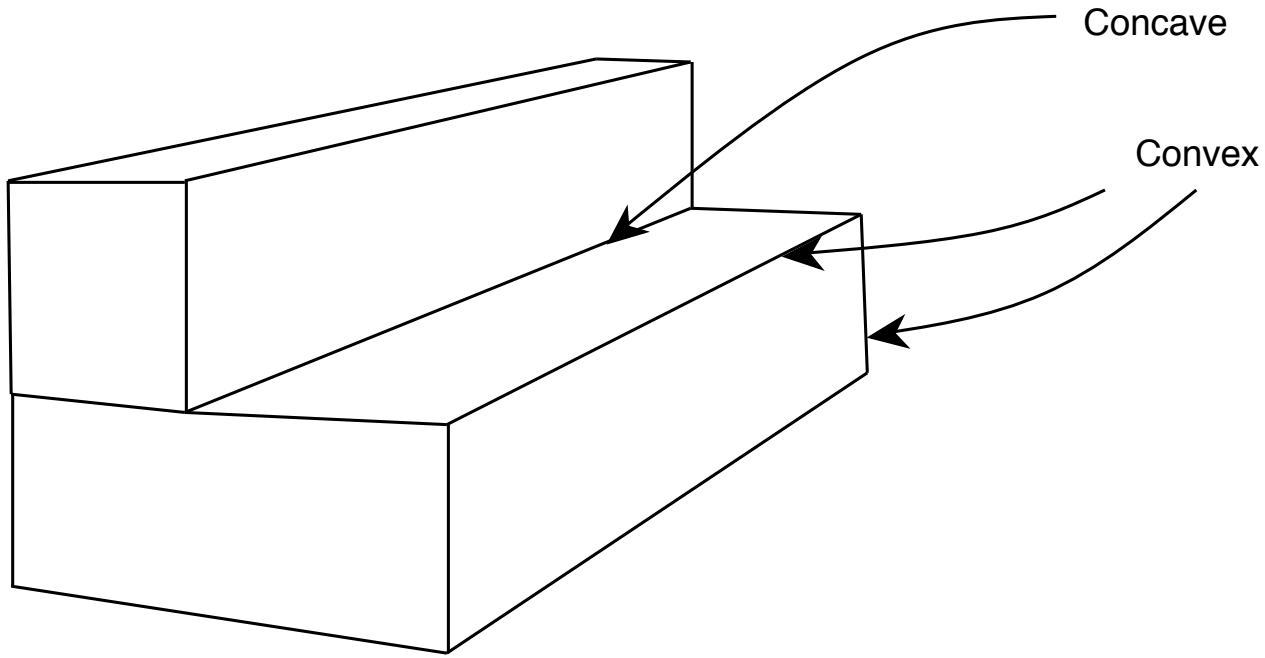
- N-queens problem: Place n queens on an $n \times n$ chessboard so that they don't attack one another



(From okpanico.files.wordpress.com)

Examples

- Computer vision: Classify edges in an image as convex or concave



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$$\begin{aligned} 3x + 4y + 6z &= 3 \\ 4x + 6y - 3z &= 4 \\ 7x - 3y - 4z &= 10 \end{aligned}$$

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- BSAT: Is a sum-of-products binary expression satisfiable, and if so, with what T/F assignments?

$$ABC + \overline{A}\,\overline{B}\,C + \cdots + ABC\overline{D}$$

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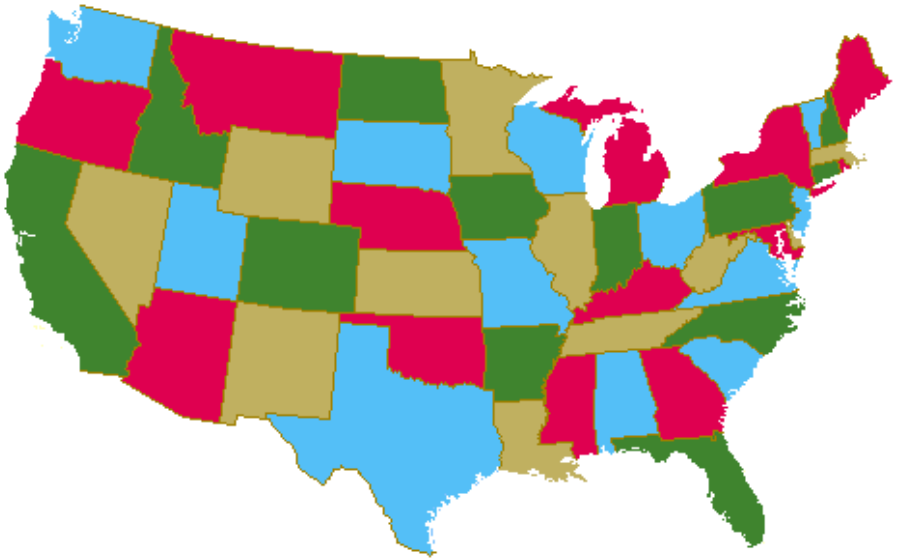
- Map coloring: Can we color a map of connected regions with *n* colors without two adjacent regions having the same color?



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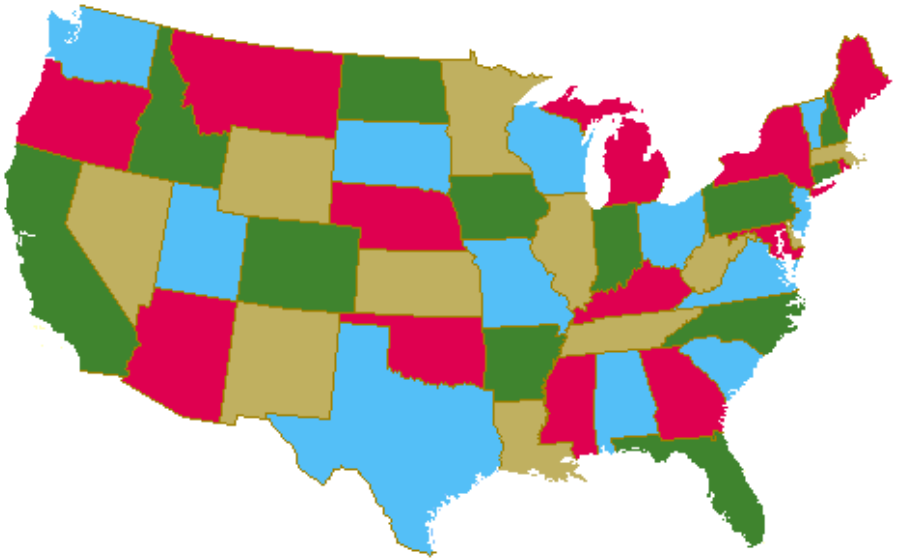

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- Scheduling: Scheduling a meeting with *n* people

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- Sometimes: radically reduce search effort

Constraint satisfaction problems

Constraint-satisfaction problems

- *Constraint satisfaction problems* (CSPs):
  - Require set of variables to be bound to values $\in$ domain
  - Require constraints to be *satisfied*
- Instead of trying all possible variable/value assignments via search...
- *Propagate* constraints and values $\Rightarrow$ reduce domains of variables
- $\mathcal{O}(v^d)$ in w.c.: try $\Rightarrow \mathcal{O}(v^{d'}), d' \ll d$ in average case
- Fox, others: All problems can be reformulated $\Rightarrow$ CSPs

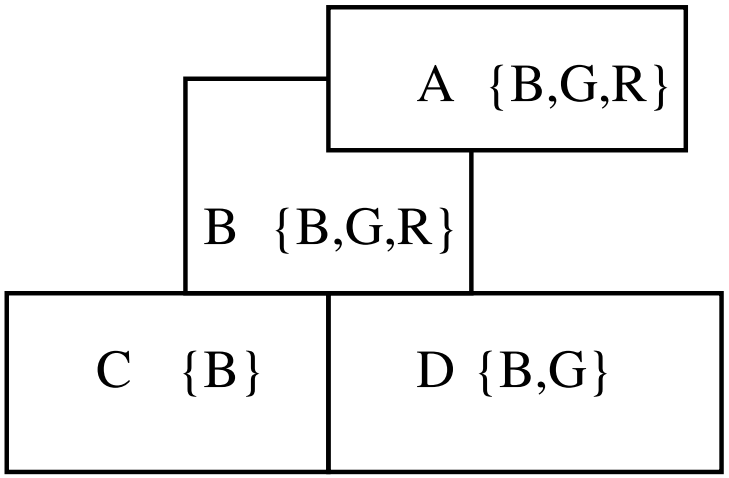
Constraints

- Types by arity:
  - *Unary constraints:* constraint on single value
  - *Binary, ternary, n-ary constraints:* restrict value of variable depending on value of other variable(s)
  - All $n - ary$ constraints can be $\Rightarrow$ binary constraints
- Types by whether absolute or preference constraints

Constraint propagation

- Suppose we reduce domain of v_1 that constrains v_2
- Some values in v_2 might now be eliminated
- Thus decision at v_1 *propagates* via the constraint to v_2
- Propagation continues from v_2, etc.
- May ultimately change v_1 again
- Stop when no more changes occur
- More constrained the values $\rightarrow$ faster to a solution

An Example of constraint propagation



Map-coloring Problem

- Cannot color adjacent areas with the same color
- Some areas may have unary constraints which limit their domains
- Eliminate impossible assignments by propagating constraints

CSP formalism

Constraint satisfaction problem

- *Constraint graph*

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- Goal: All variables instantiated, no violated constraints

What is a state?

- State representation 1: *complete assignments*
  - Start: Graph + random assignments
  - Operator: Change variable's value
  - Goal: All constraints satisfied
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    - Set variable, check for goal
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    - E.g.: For 4-coloring of 48 states: ~10^{90}
- State representation 2: *partial assignments*
  - State: Graph + domains – singleton = assignment
  - Operator: Make assignment
  - After each assignment: propagate constraints
  - Goal: all singleton domains
  - Encounter empty domain: backtrack
  - Systematically explore space by choosing how vars instantiated

Constraint Propagation

Constraint propagation

- After value selected, propagate effects using constraints
- Propagation $\Rightarrow$ narrowing of domains to be consistent
- Two types of consistency:
  - Node consistency
  - Arc consistency

Node consistency

- Check unary constraints
- Pre-processing, $\mathcal{O}(n)$ step
- E.g.:
  - Map-coloring problem:
    $\forall\, v$ dom$(v) = \{red, green, blue, yellow\}$
  - Texans object to blue, dom(Texas)
    $= \{red, green, yellow\}$
- All unary constraints satisfied: graph is *node-consistent*
- Unary constraints reduce |domain| $\Rightarrow$ prunes search tree, $\downarrow$ branching factor

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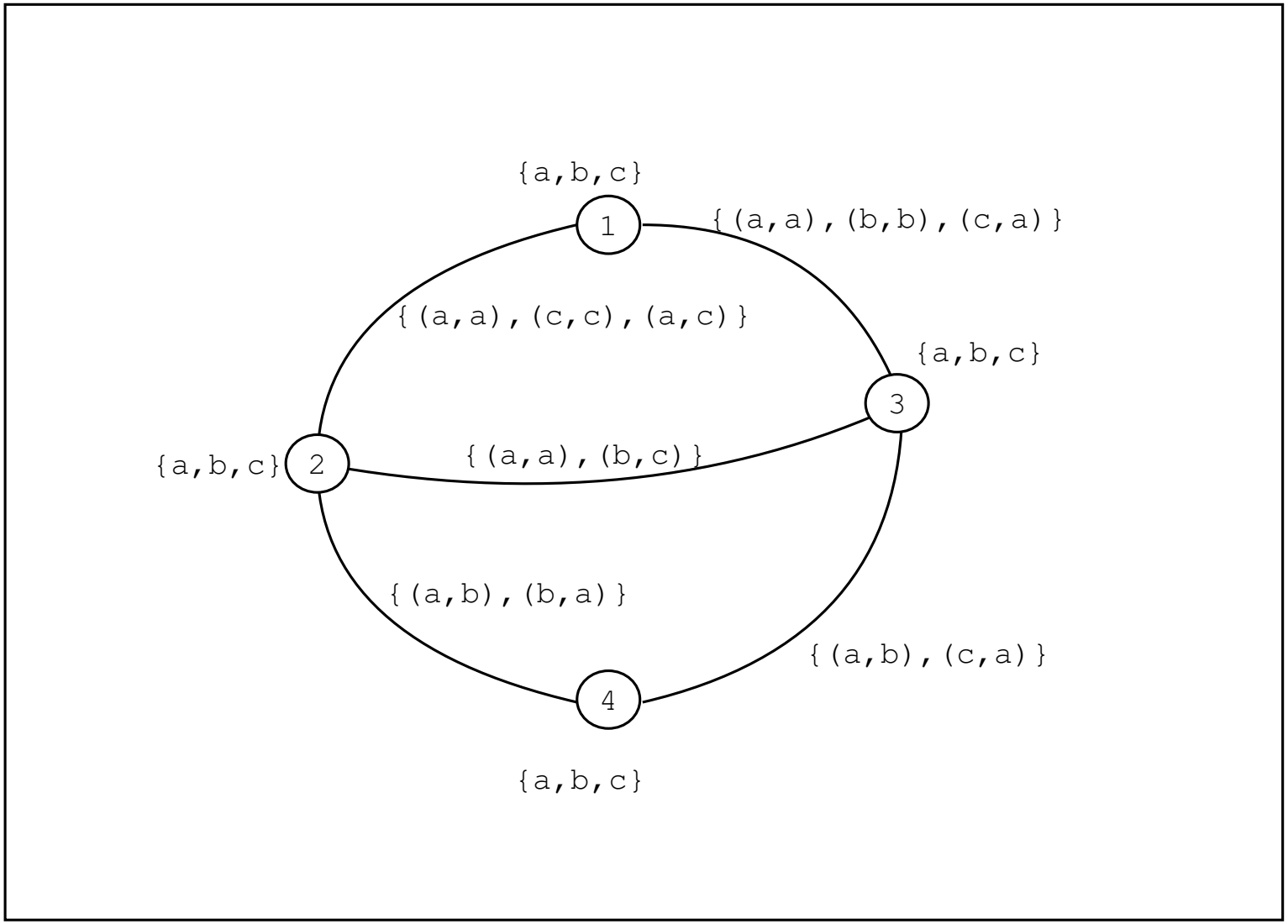
  - Initiated when variable assigned value

Example



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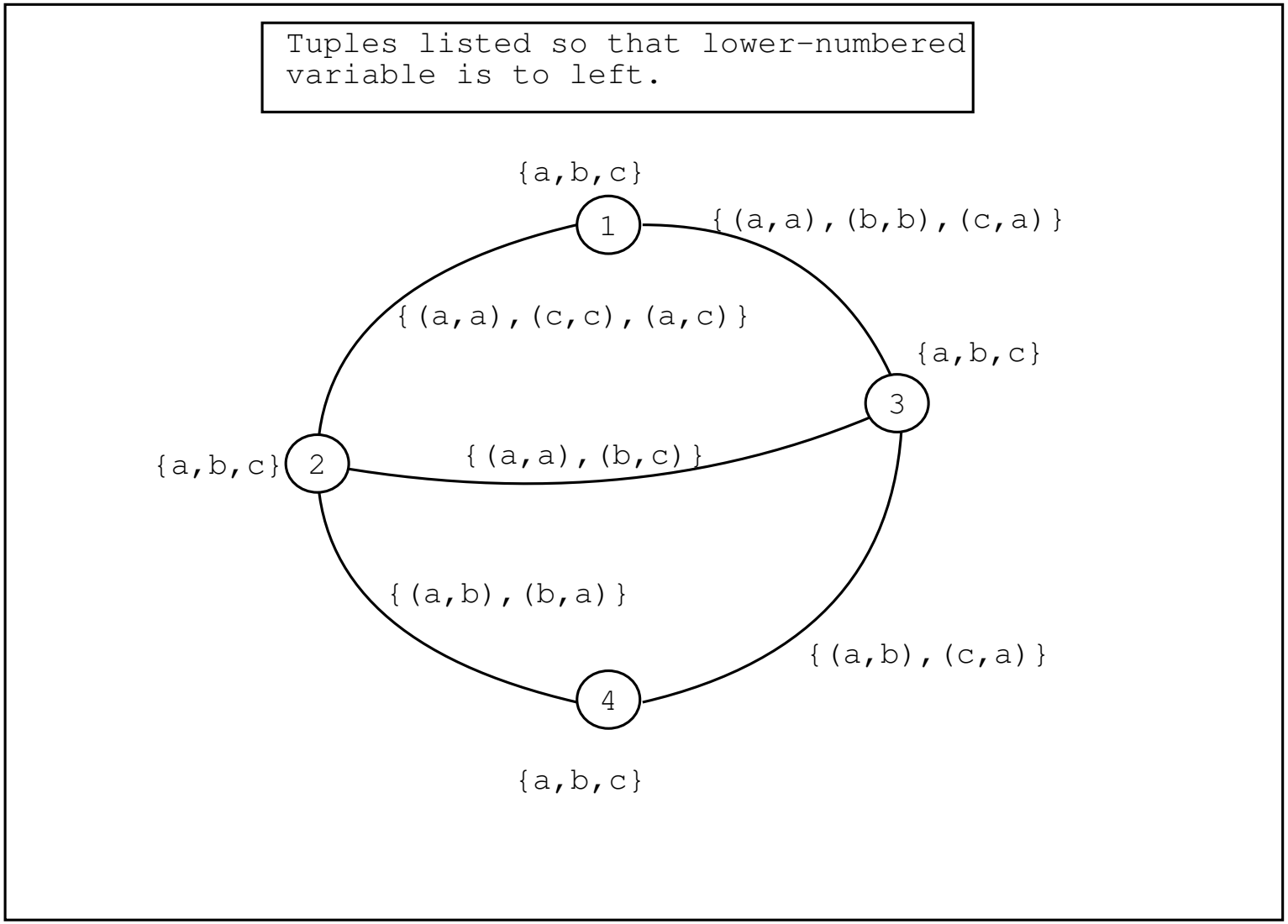

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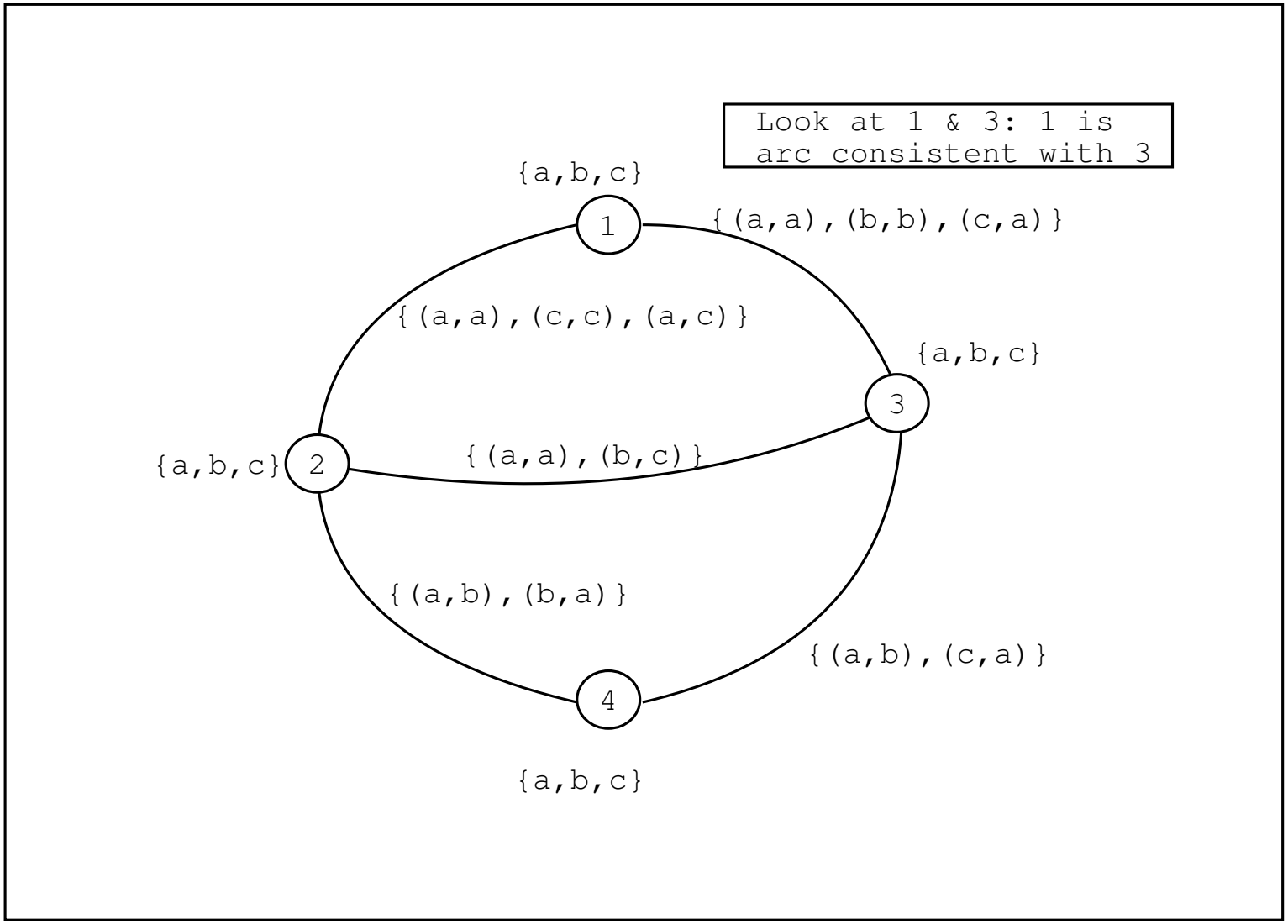

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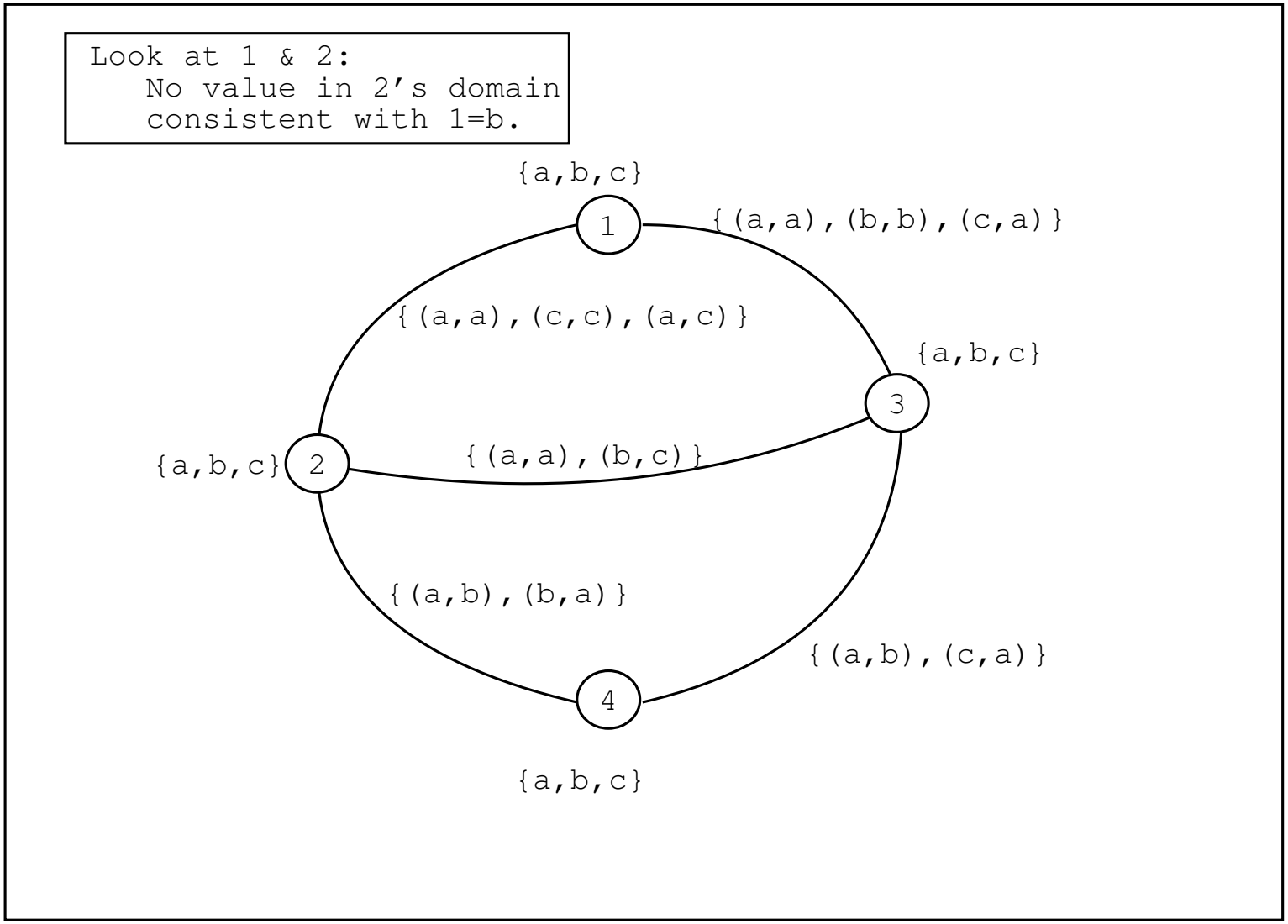

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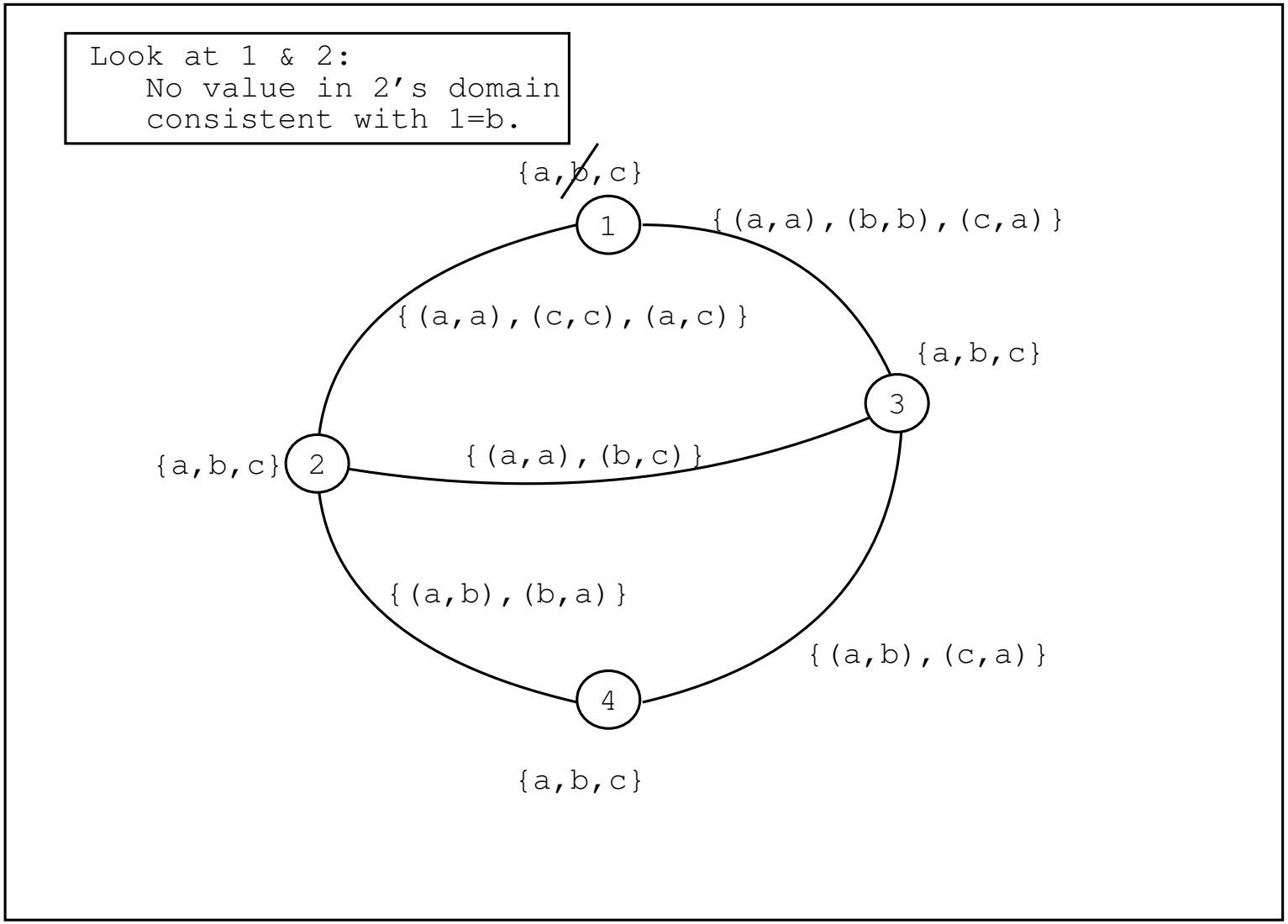

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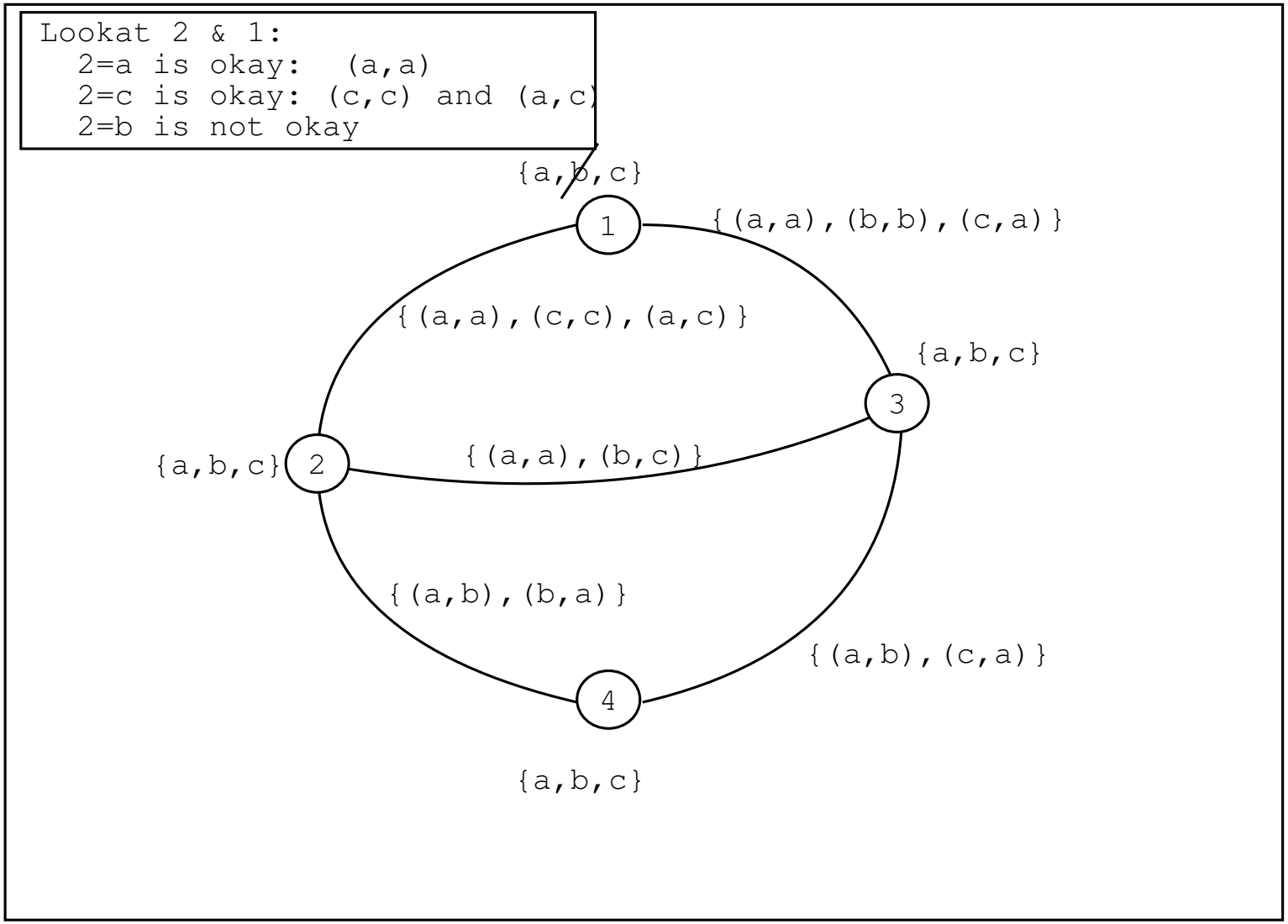

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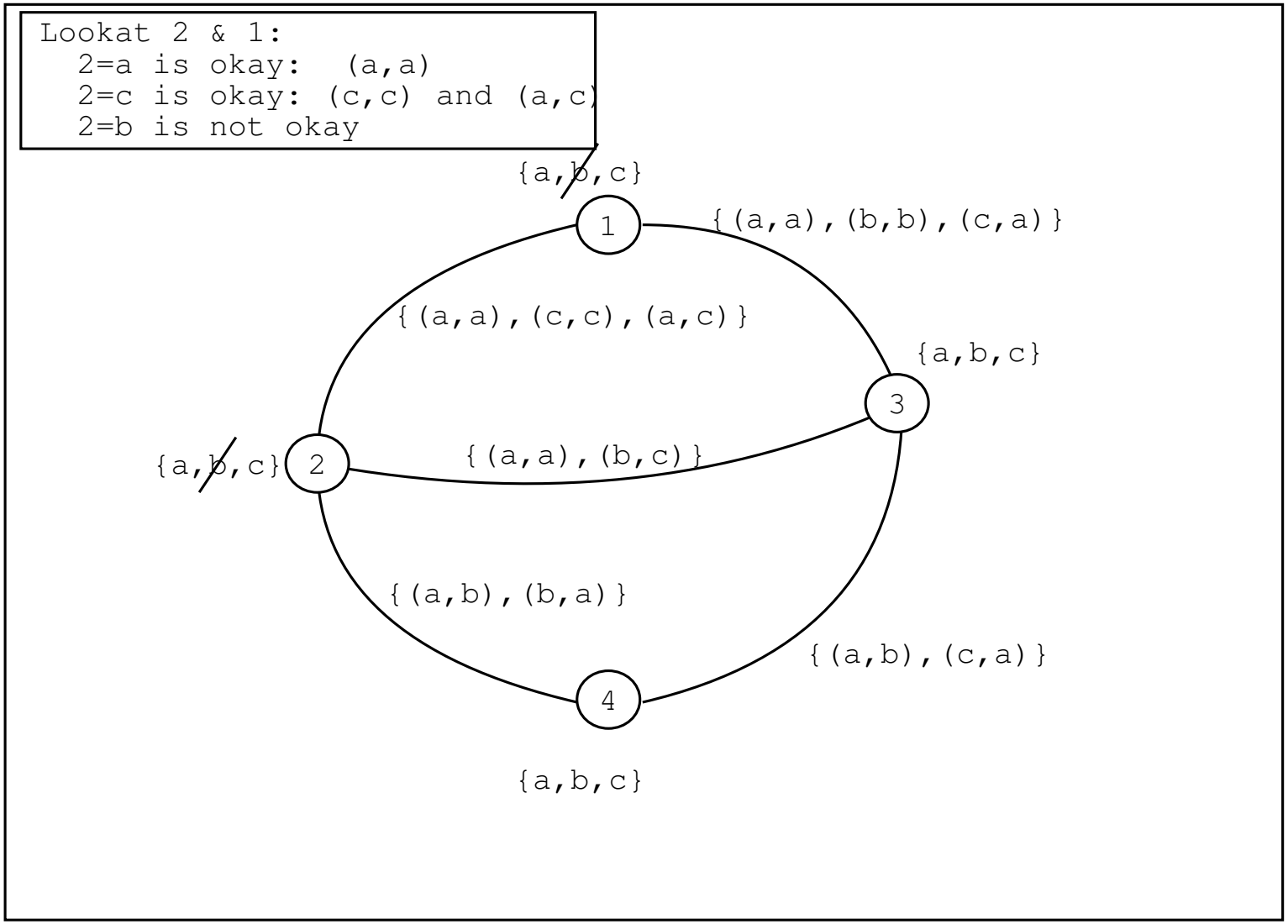

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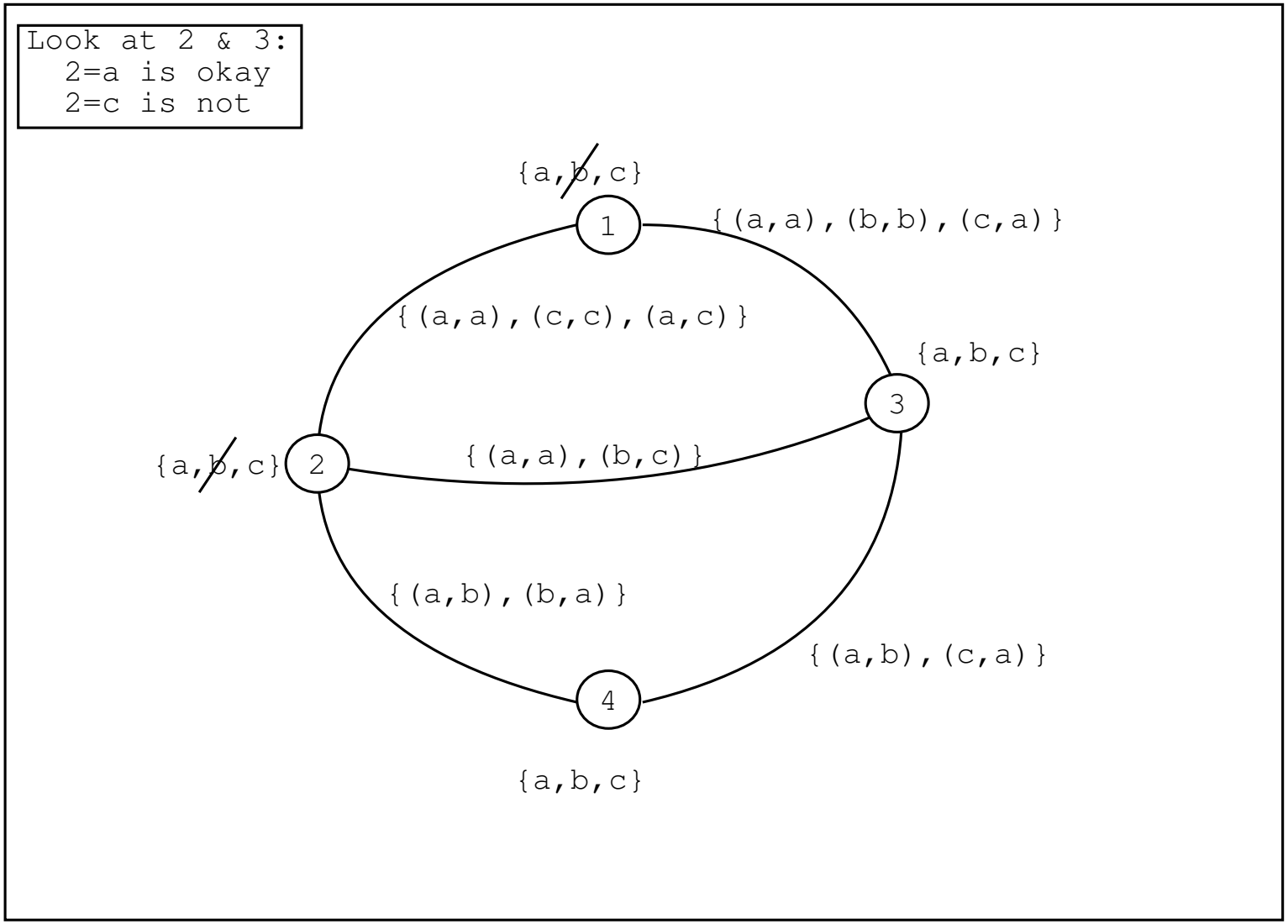

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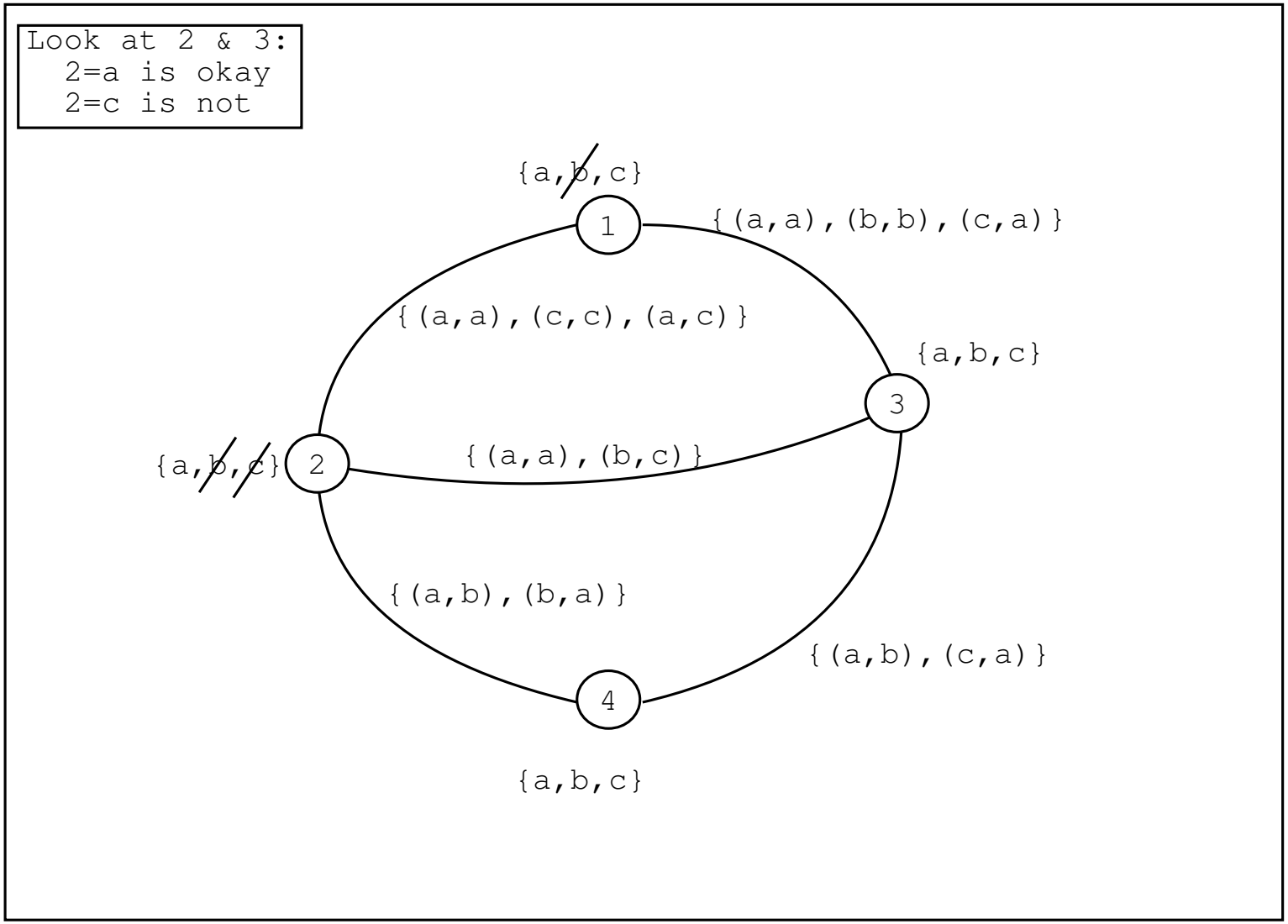

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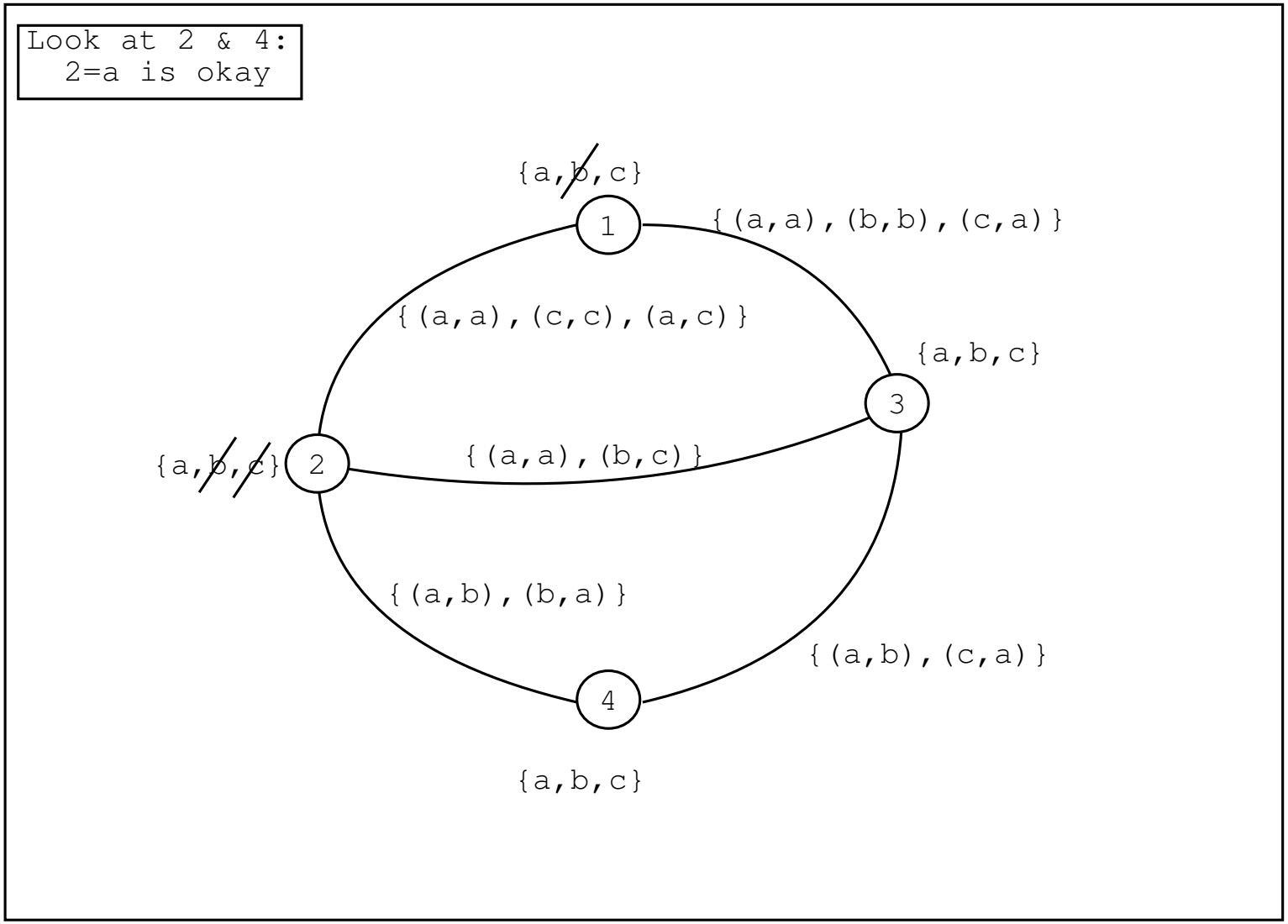

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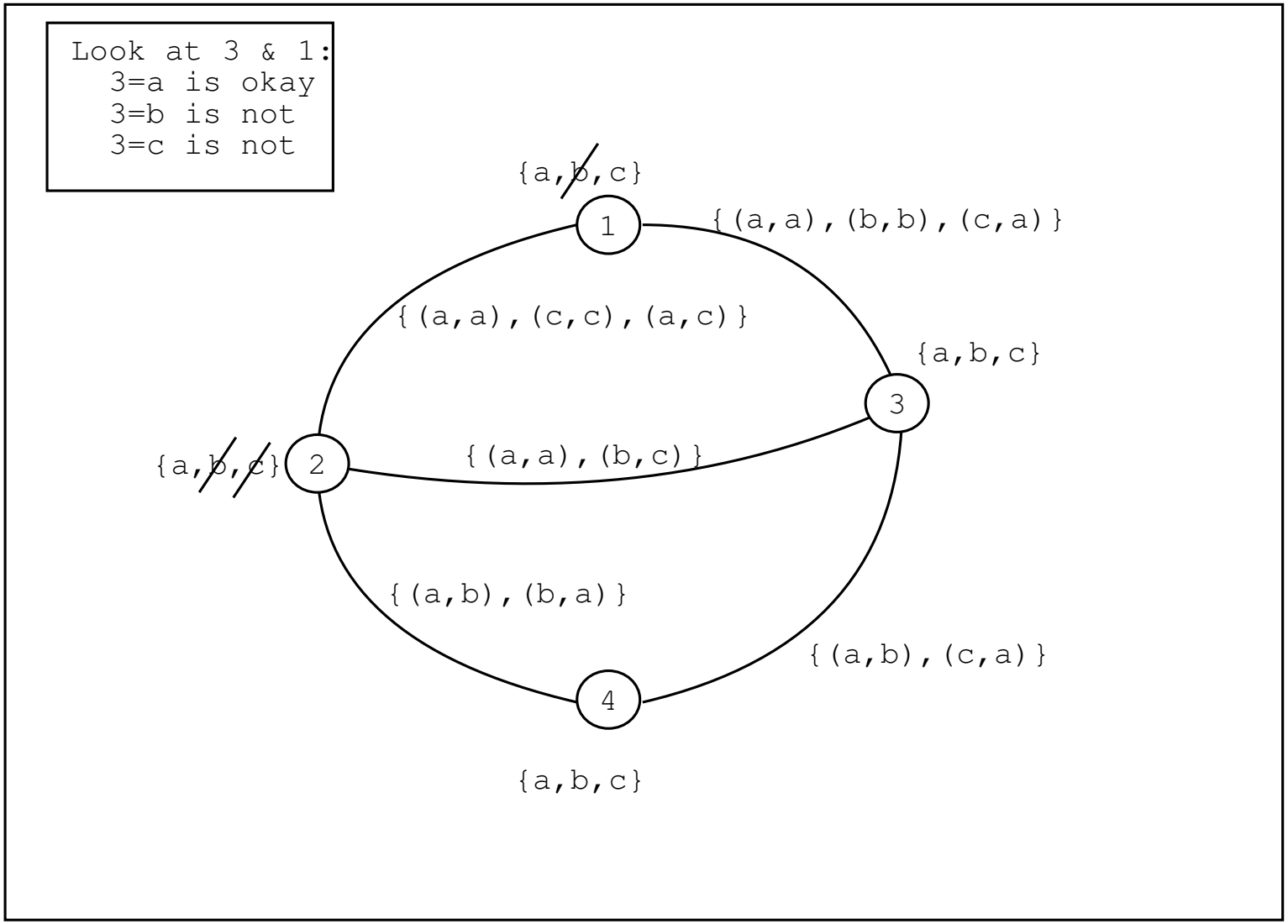

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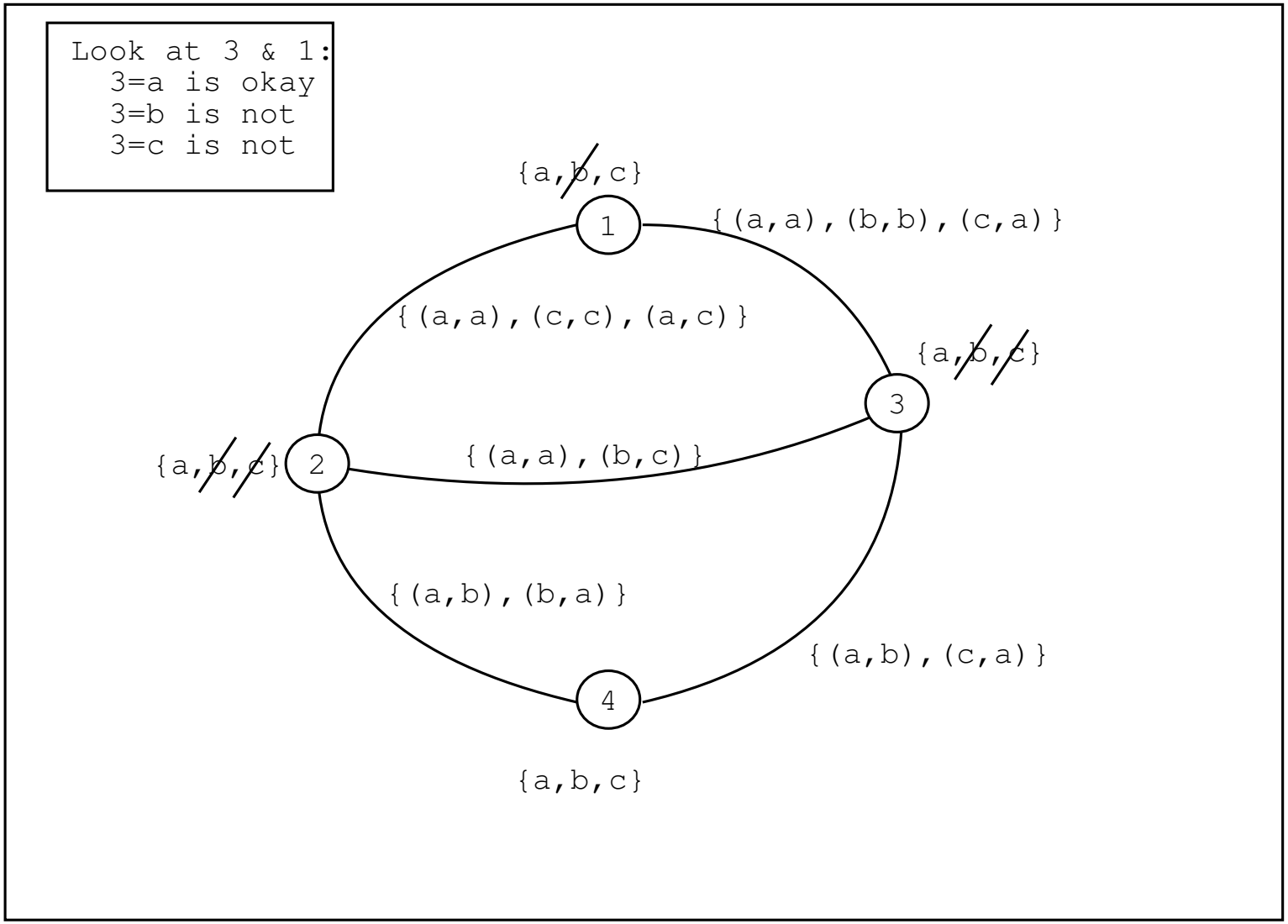

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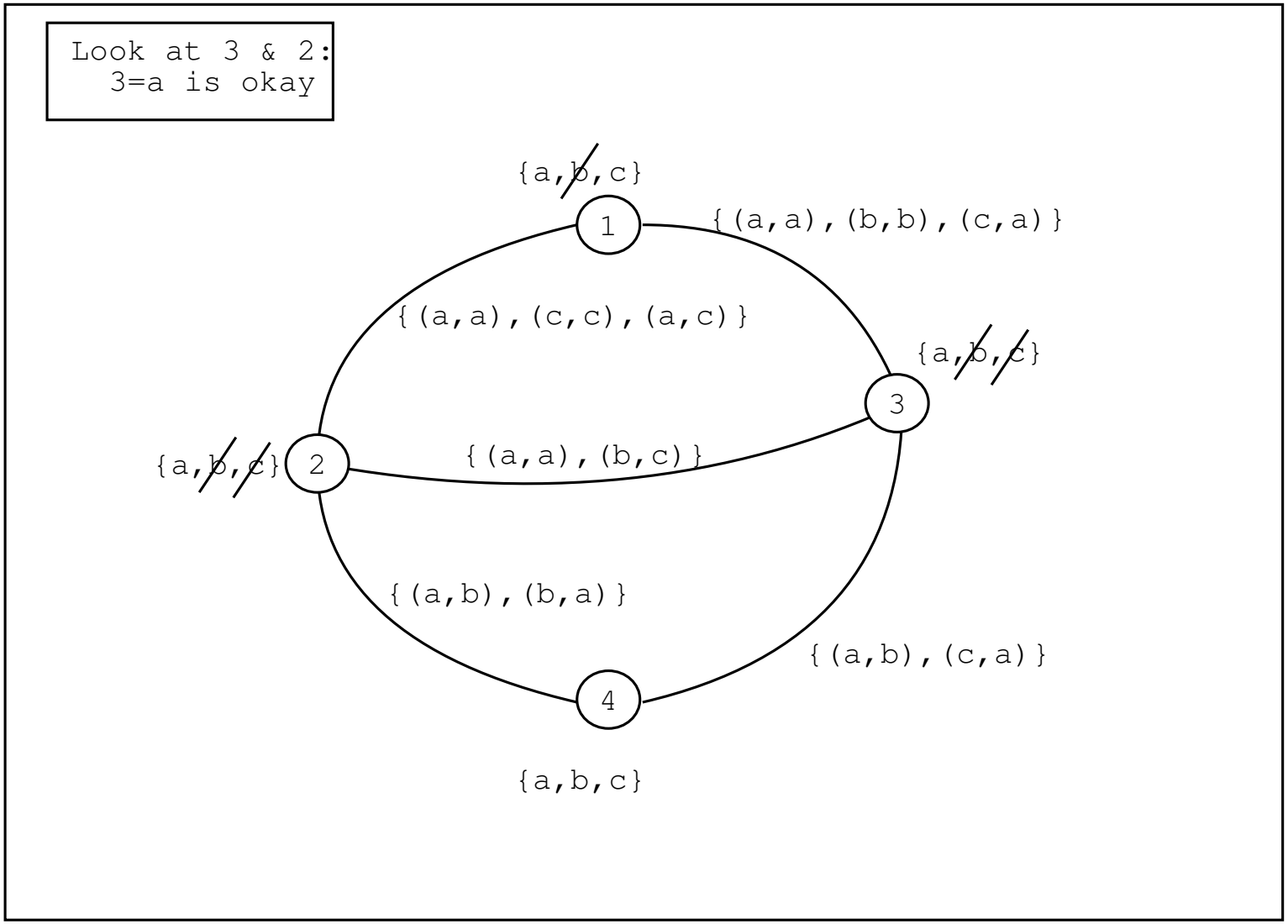

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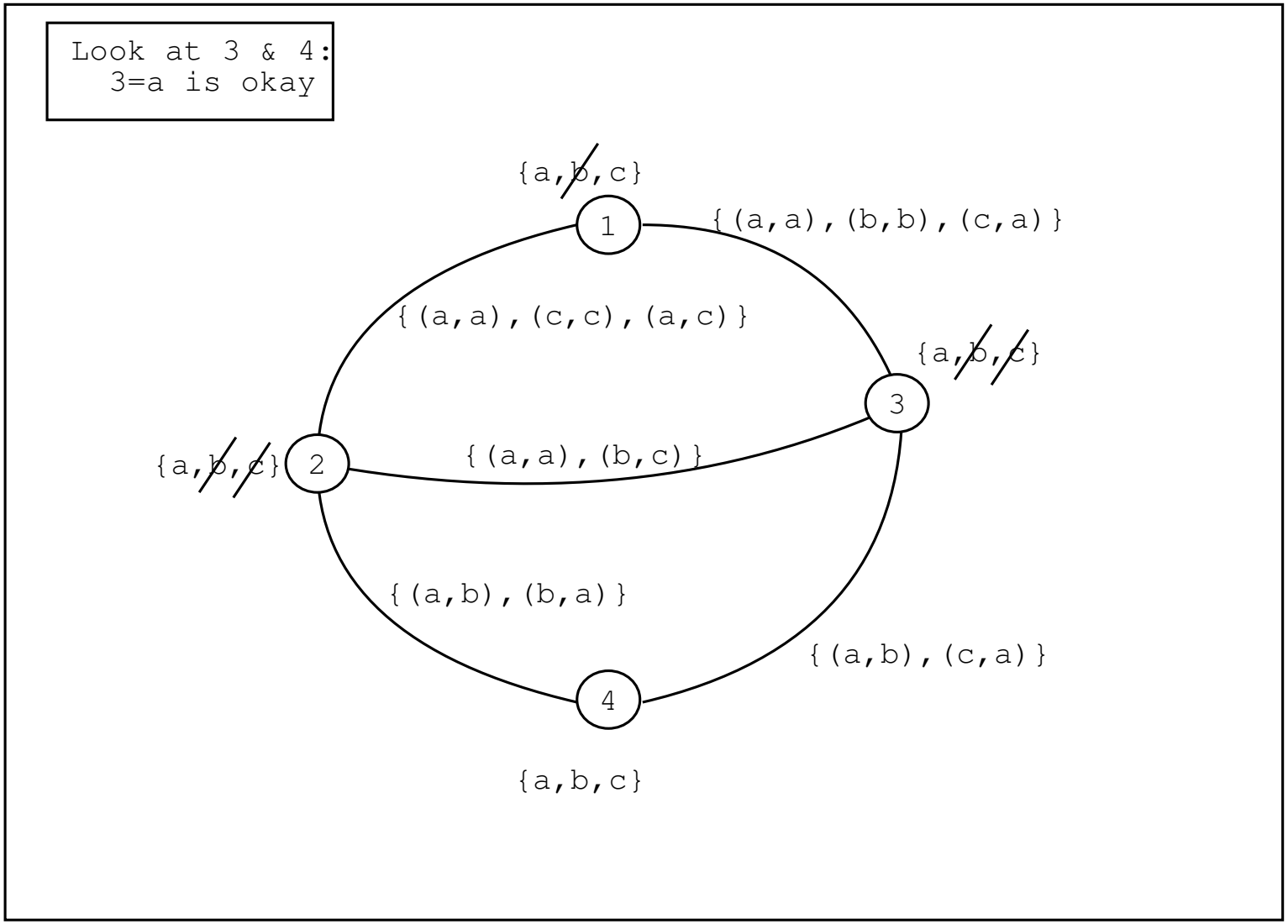

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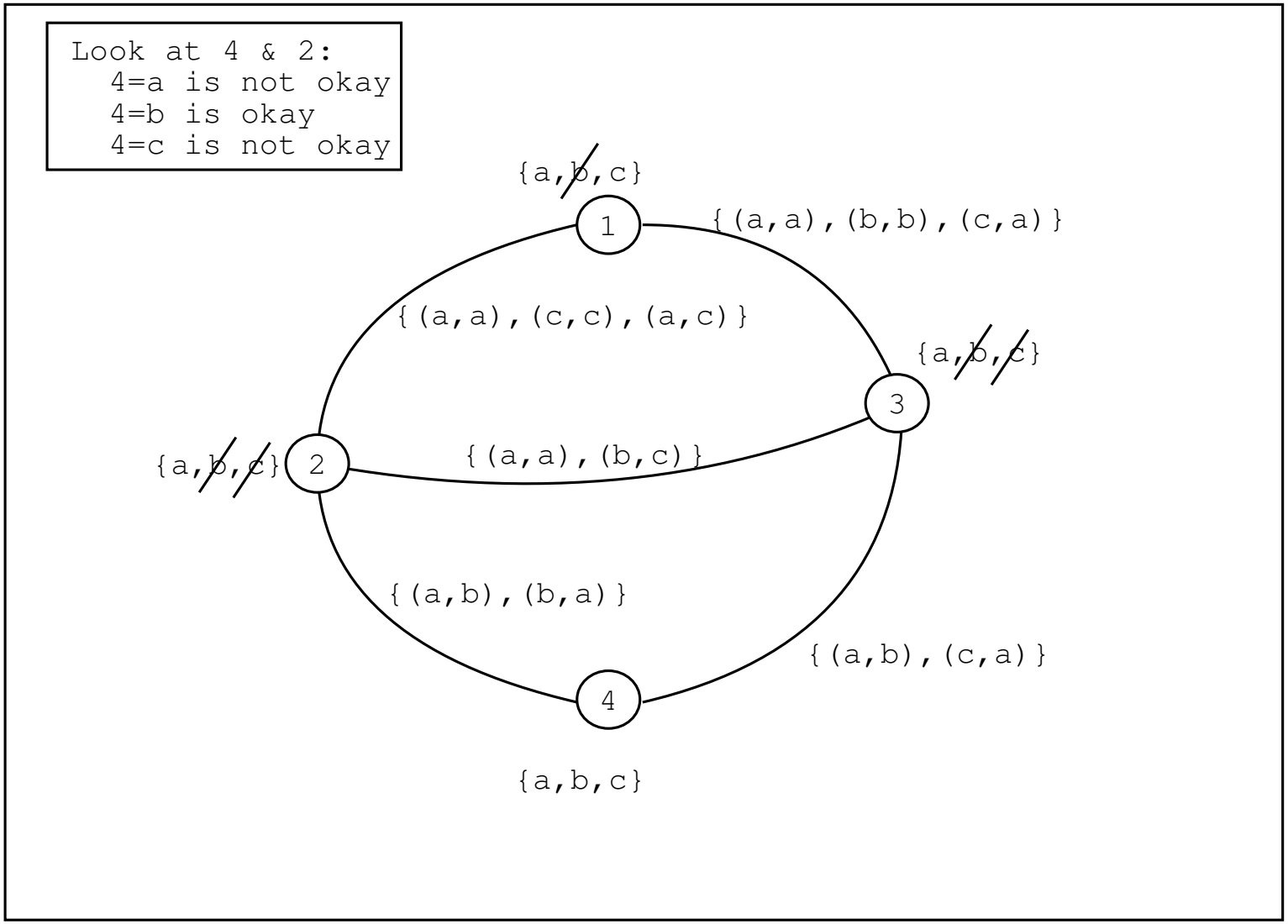

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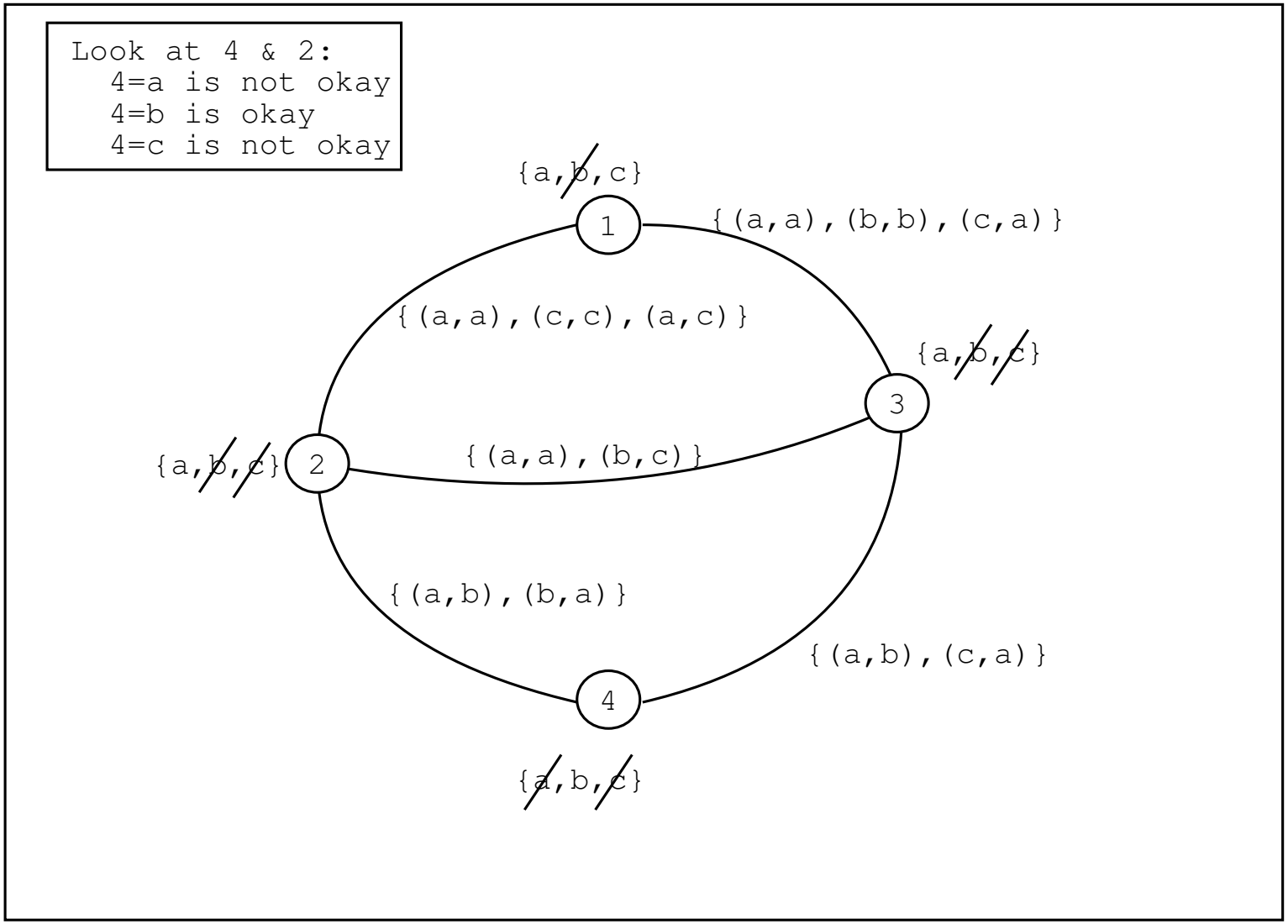

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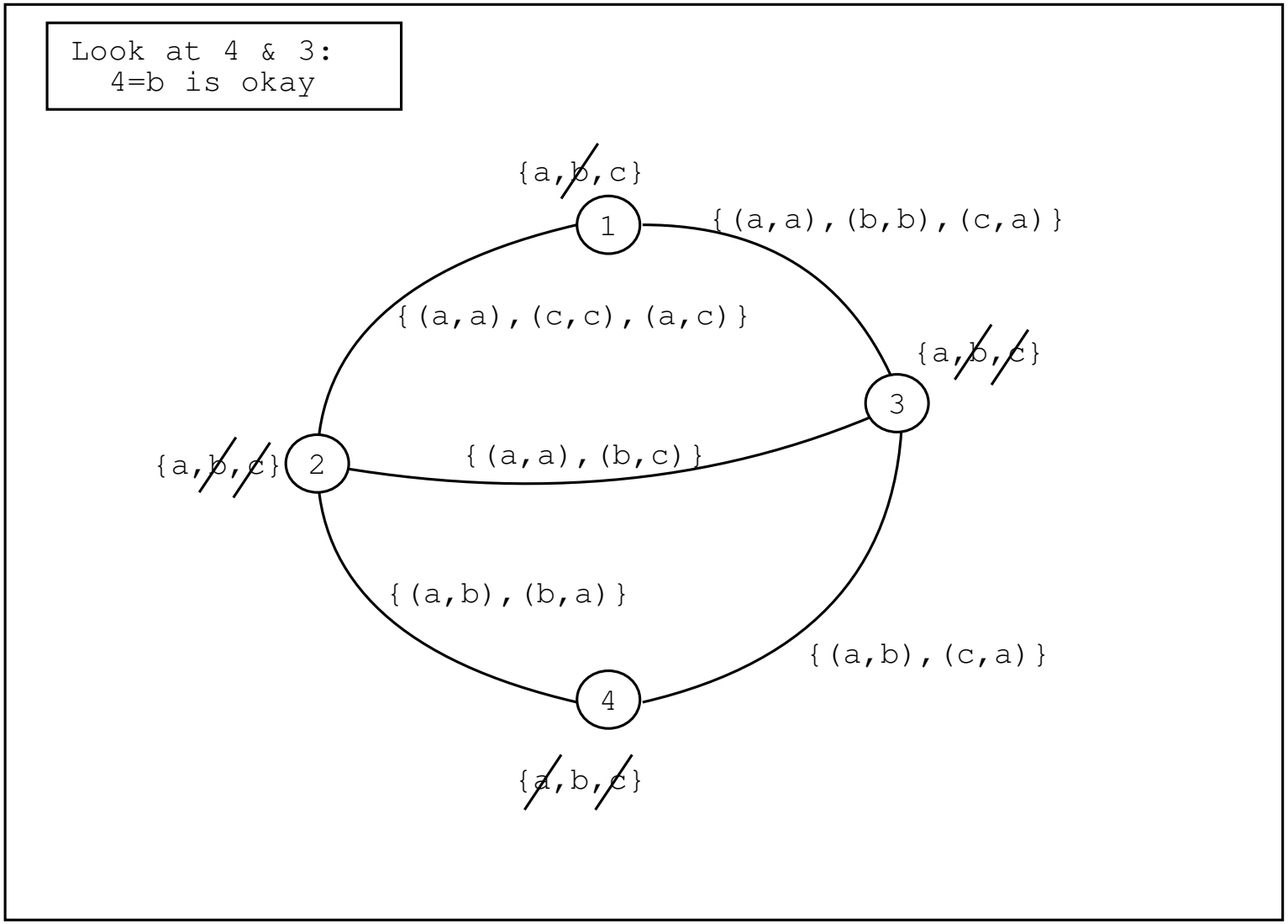

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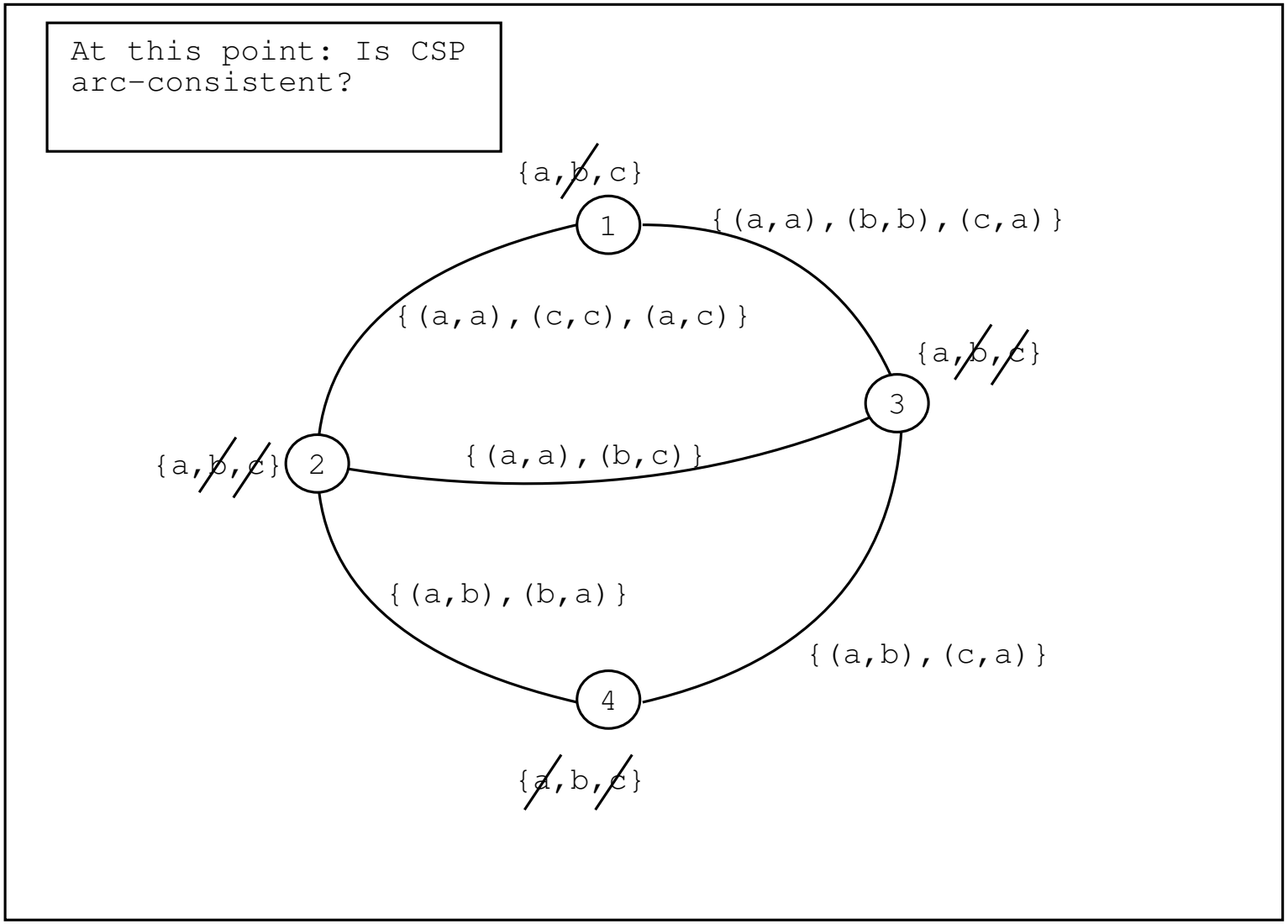

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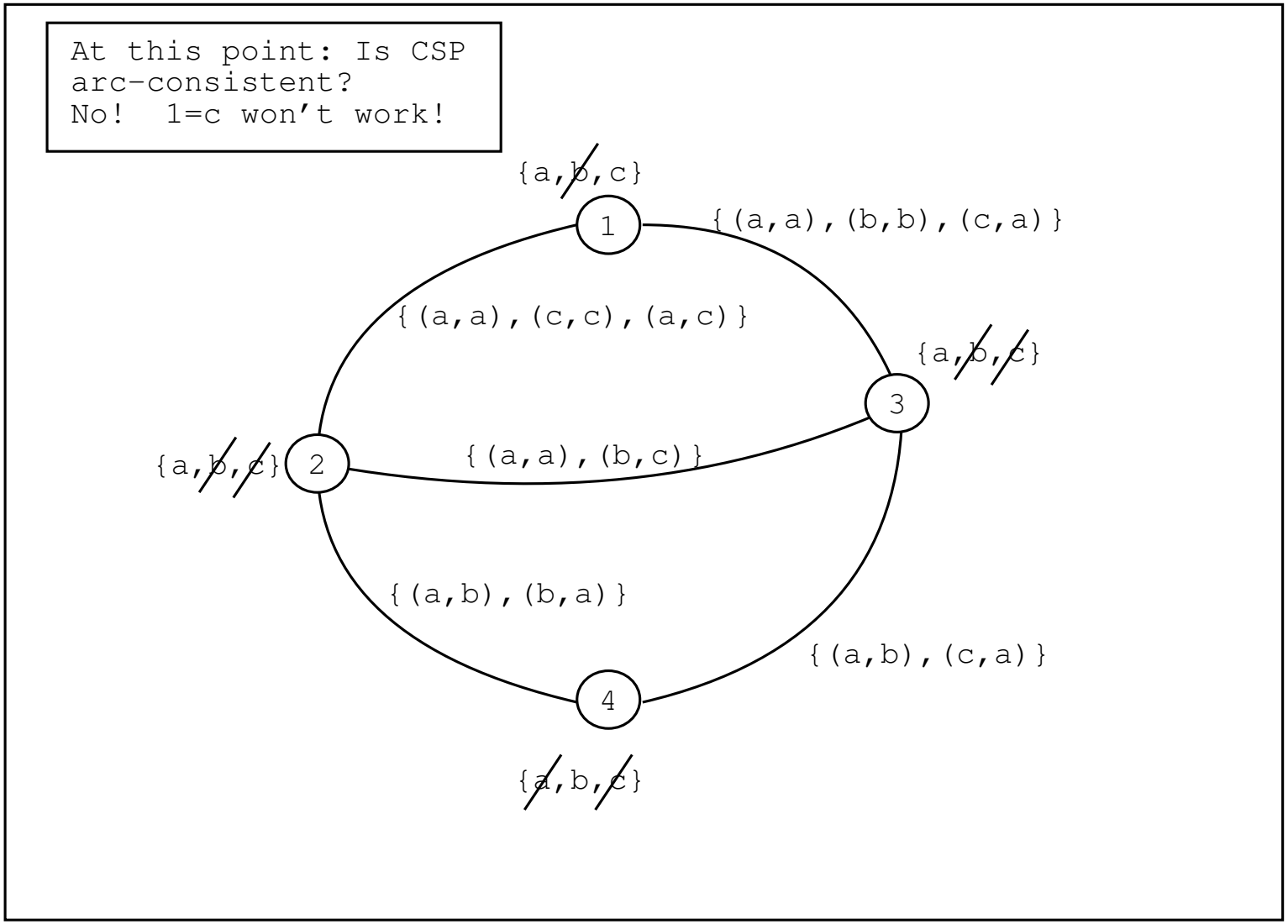

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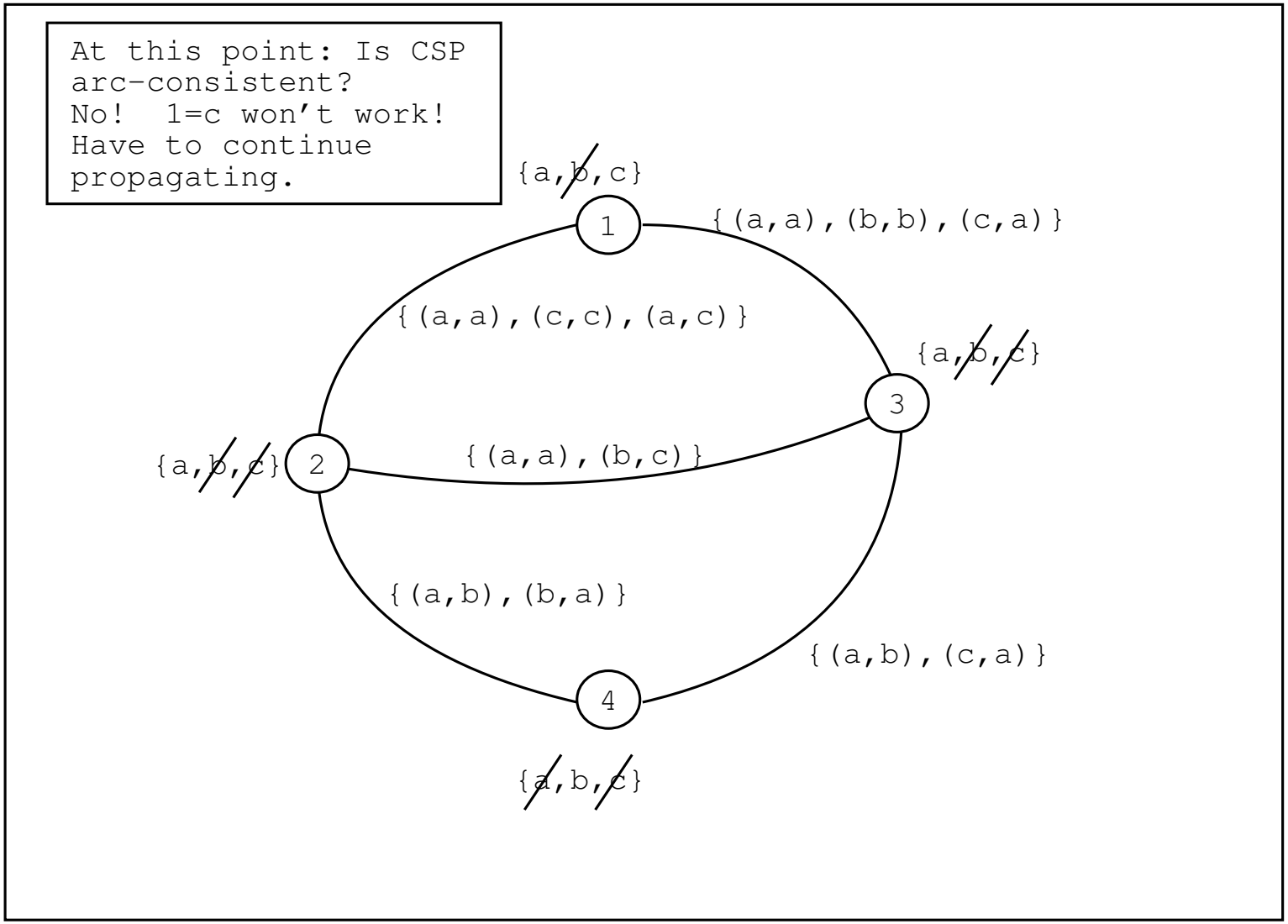

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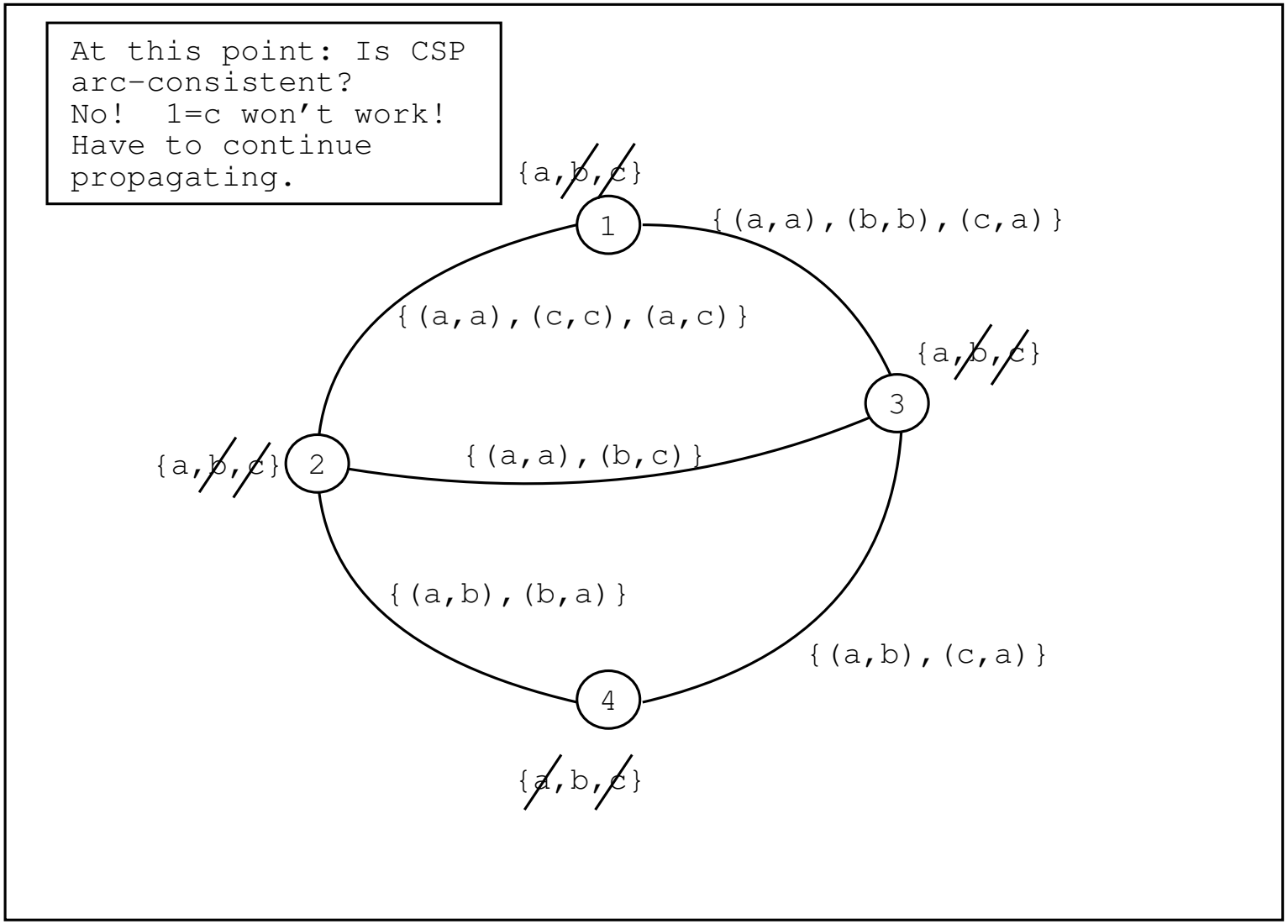

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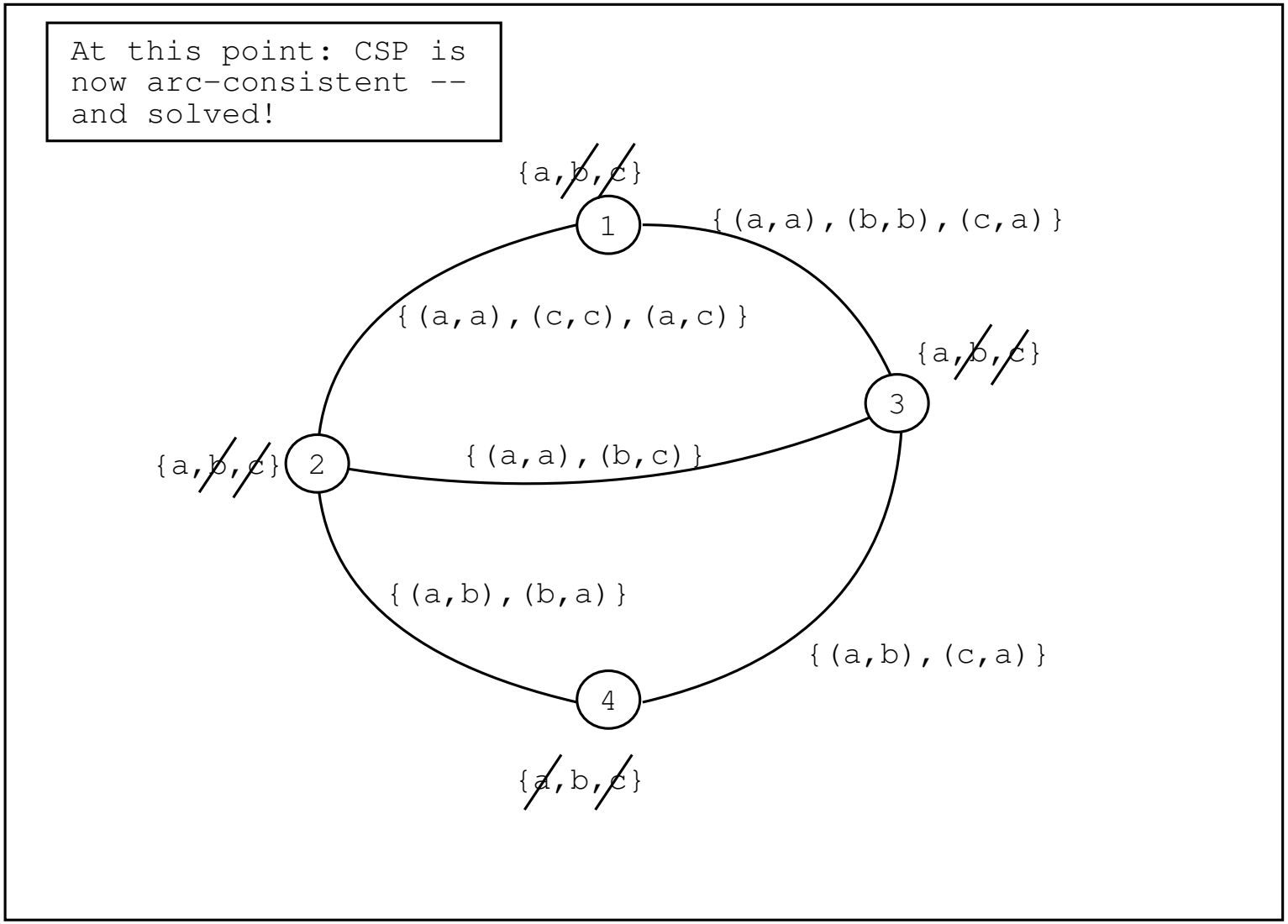
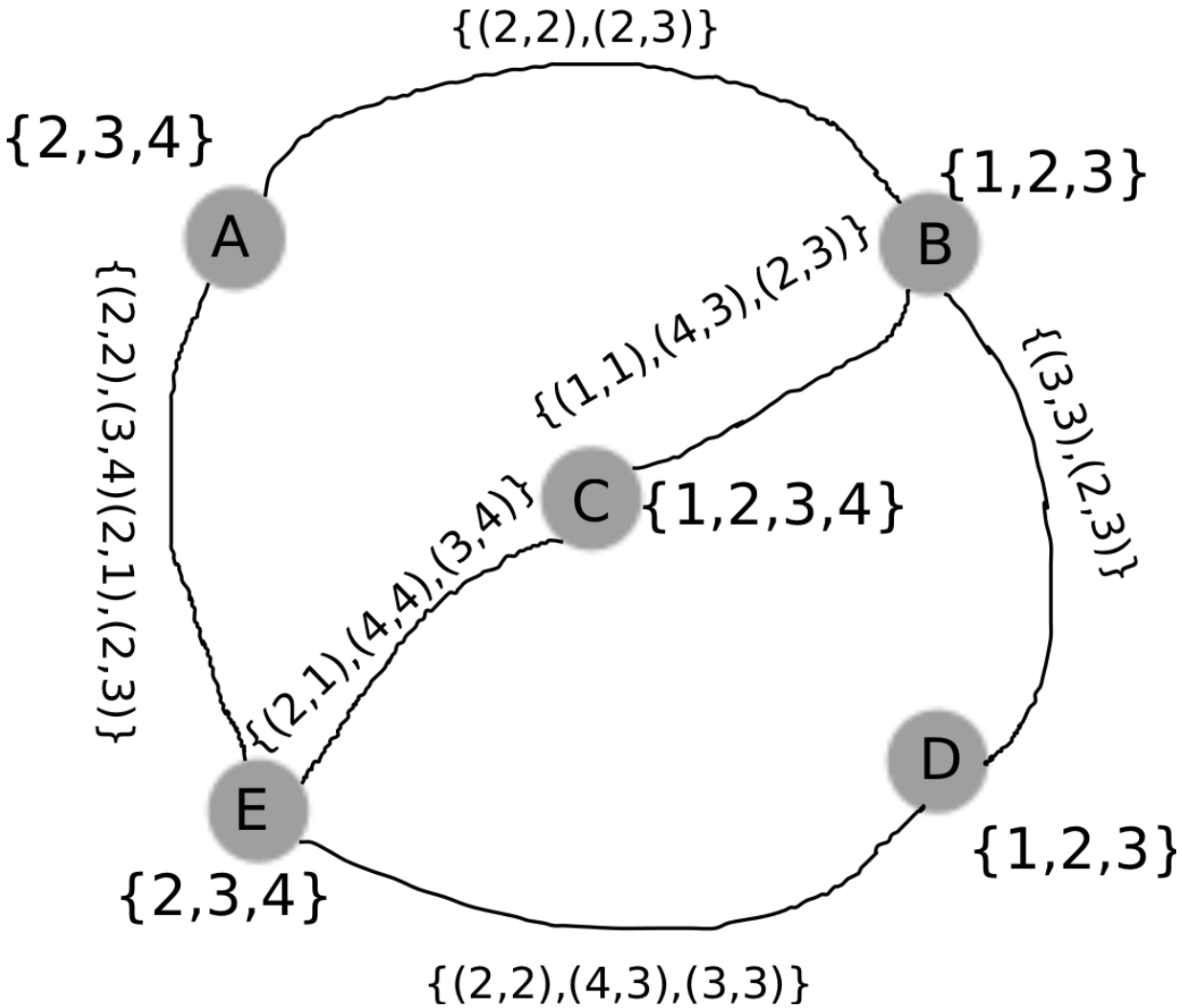
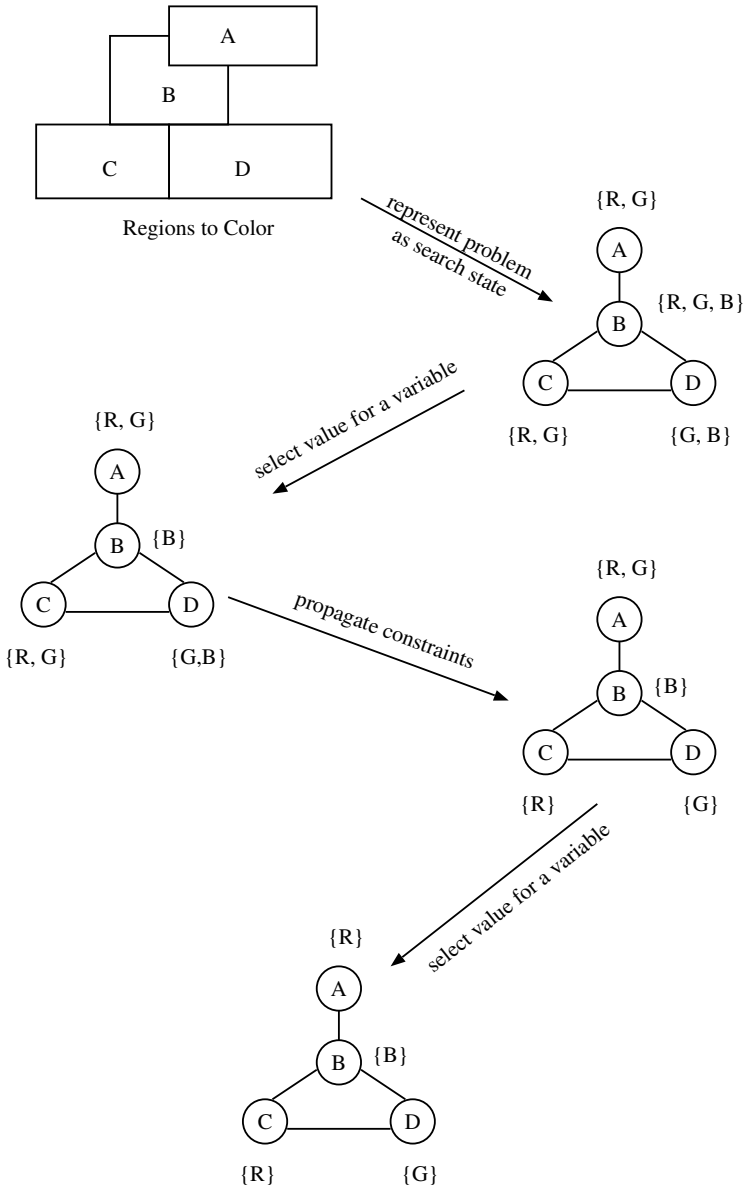


Your turn



Example of CSP



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CSP as search

- Best case: value selection + propagation $\rightarrow\rightarrow$ solution
- But it's a search process :
  - But what if dead-end?\ Backtrack
  - And *which* variable, *which* value to pick each choice point?

Heuristics for CSP search

- Which variable to set?

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- Which value to use?
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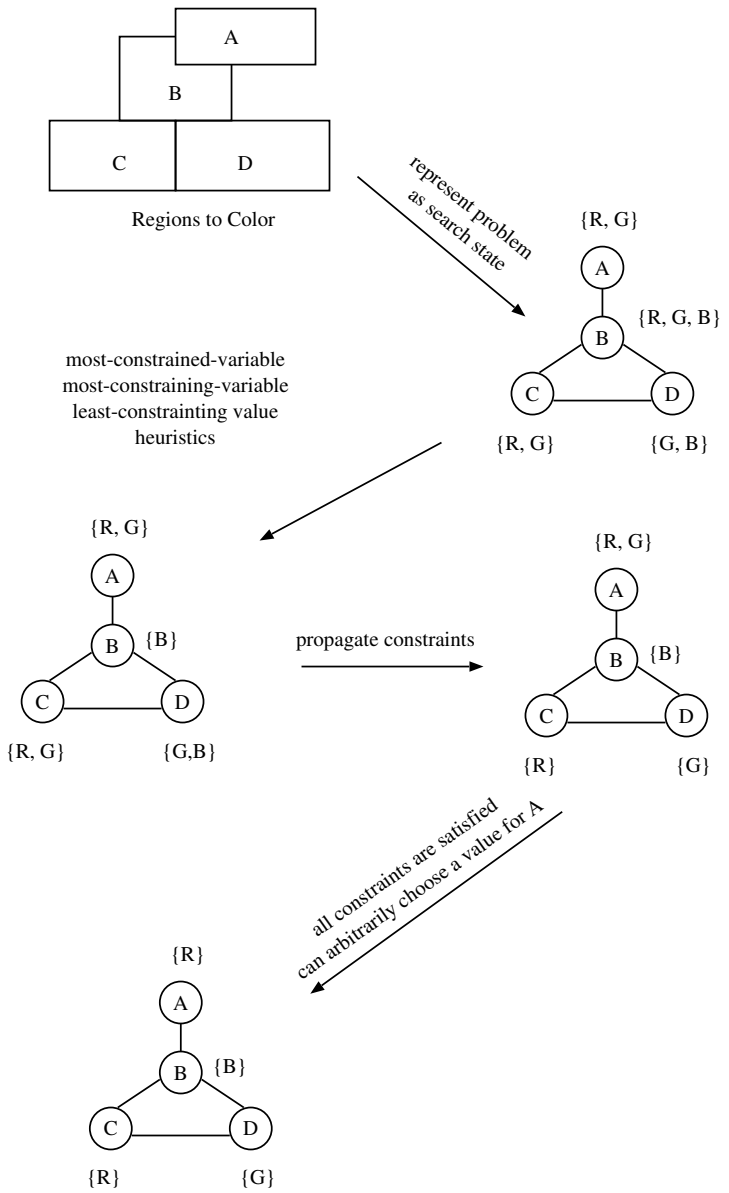
    - increases likelihood of success

Example



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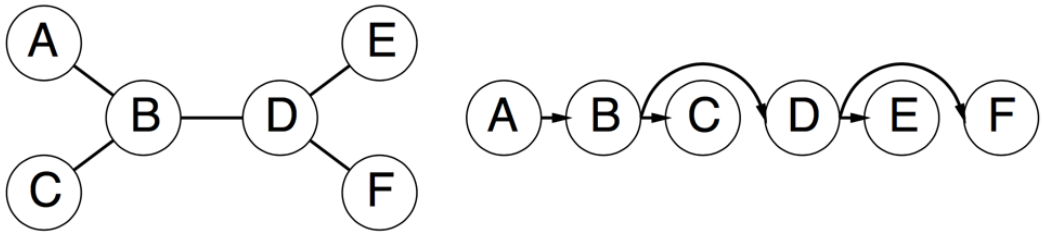

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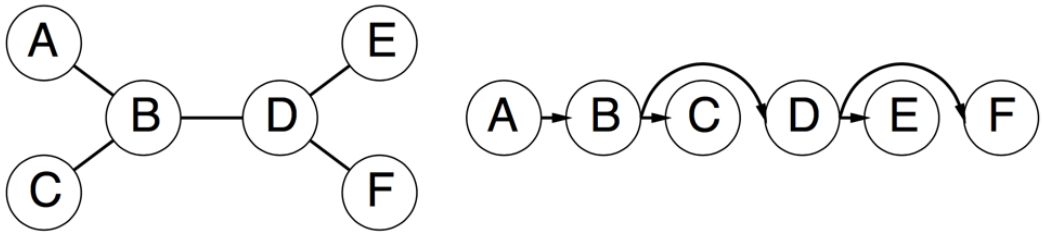

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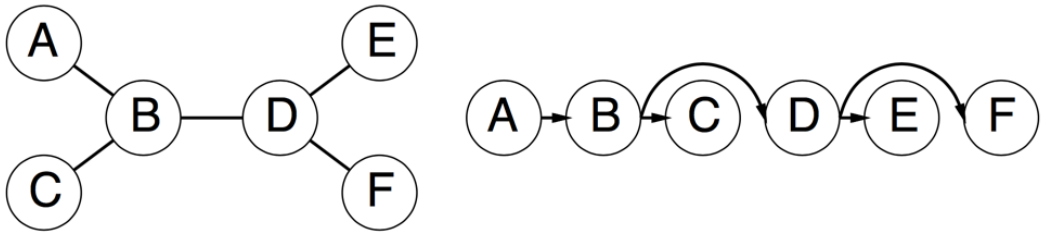

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  - If cutset size c, $\mathcal{O}(d^c \cdot (n-c)d^2)$

(From S. Russell's slides)

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  - For almost any number of queens

(From S. Russell's slides)

Miscellaneous

Continuous variables

- Many real-world problems – e.g., scheduling times for space applications, etc.
- If linear constraints: solvable by *linear programming* in polynomial time

Constrained Heuristic Search (CHS)

- Can we use even more heuristic information?
- CHS (Fox et al., 1989): Constraint graphs become states in state space search graph
- Operators: assign value, add/delete constraint, constrain domain of variable
- Heuristics: look for *textures* in graph $\Rightarrow$ operator to apply

Application: Task assignment in CoDA

CoDA

- Autonomous oceanographic sampling networks (AOSNs)

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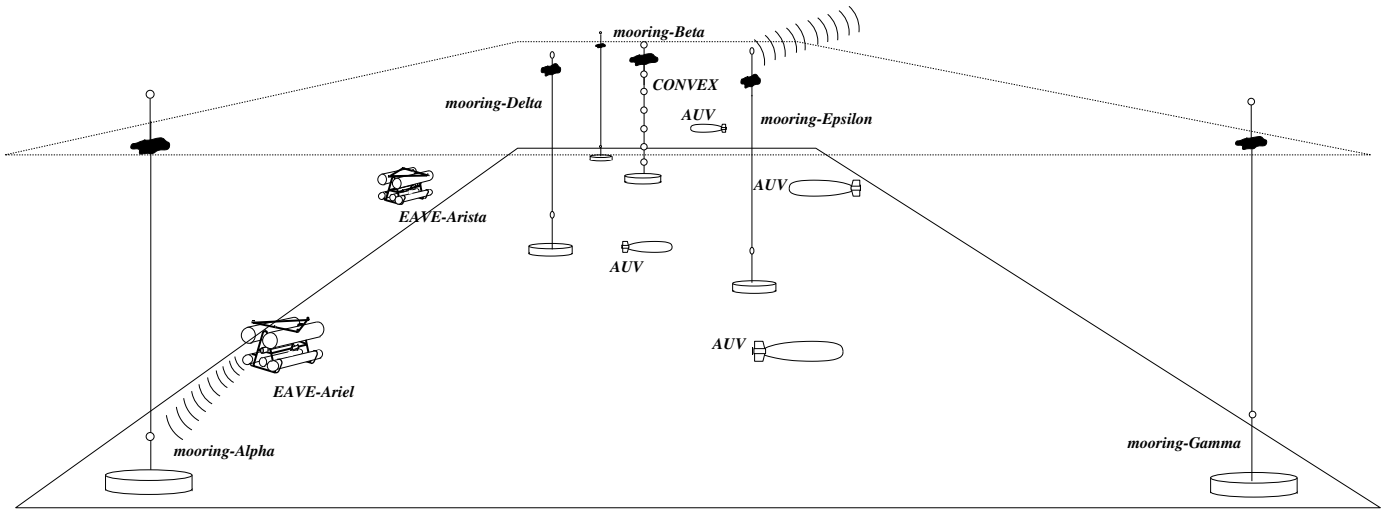

CoDA

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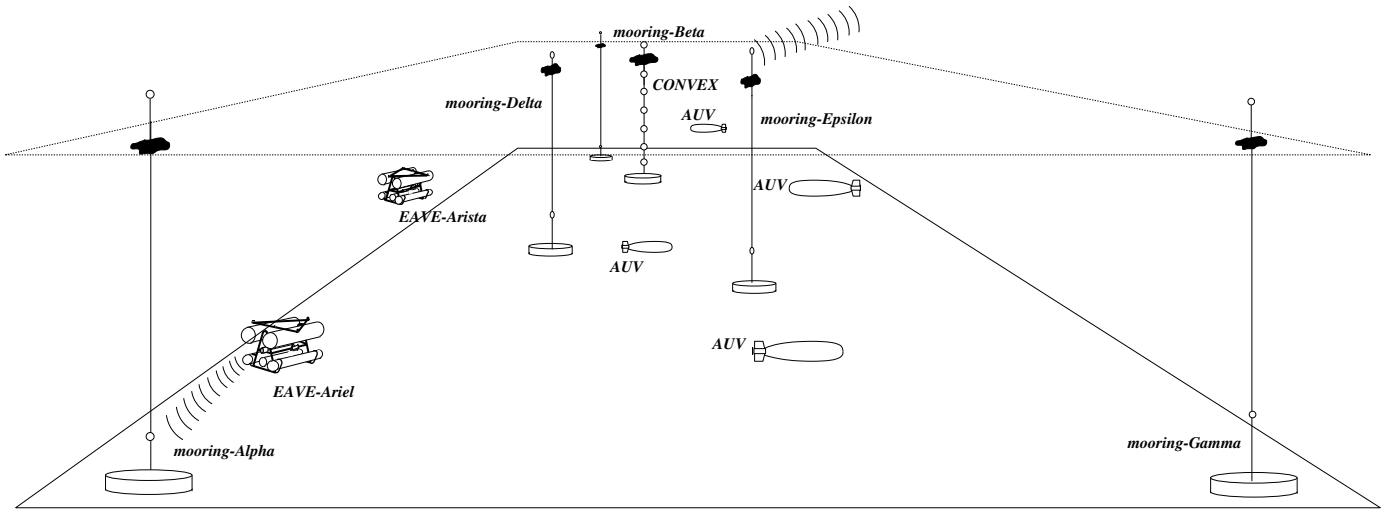

- Treat as multiagent systems (MAS): CoDA (Turner & Turner)
- Need task assignment: Constraint satisfaction problem
- Use CHS

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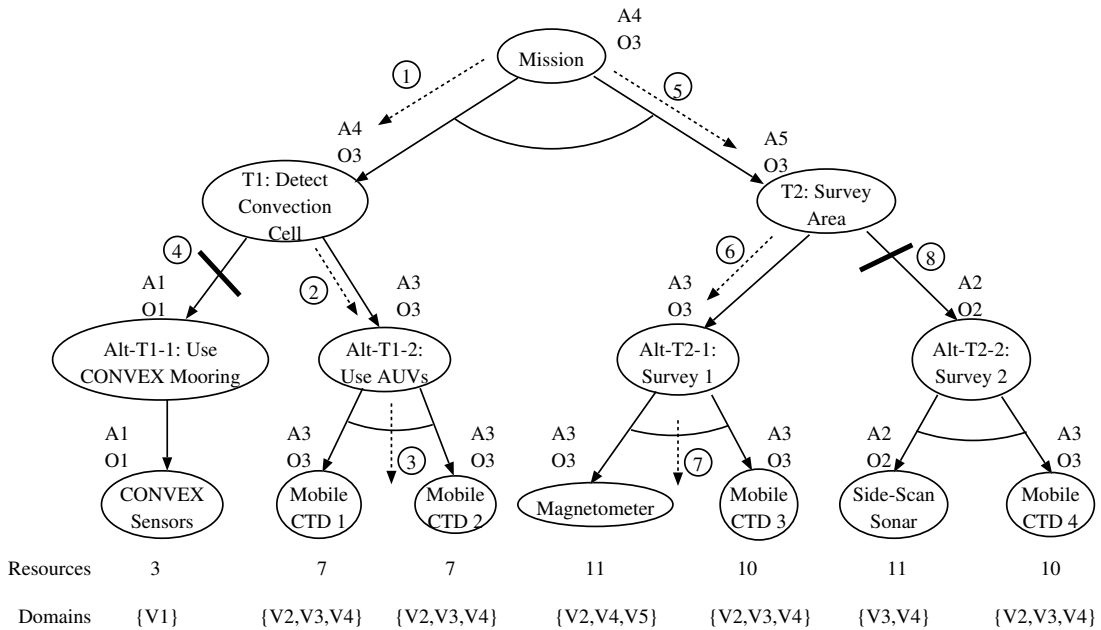

VIPs:
V1, capabilities={CONVEX sensors}, resources=5
V2, capabilities={Mobile CTD, Magnetometer}, resources=17
V3, capabilities={Mobile CTD, side-scan sonar}, resources=18
V4, capabilities={Mobile CTD, radio, camera, side-scan sonar, magnetometer}, resources=9
V5, capabilities={magnetometer}, resources=9

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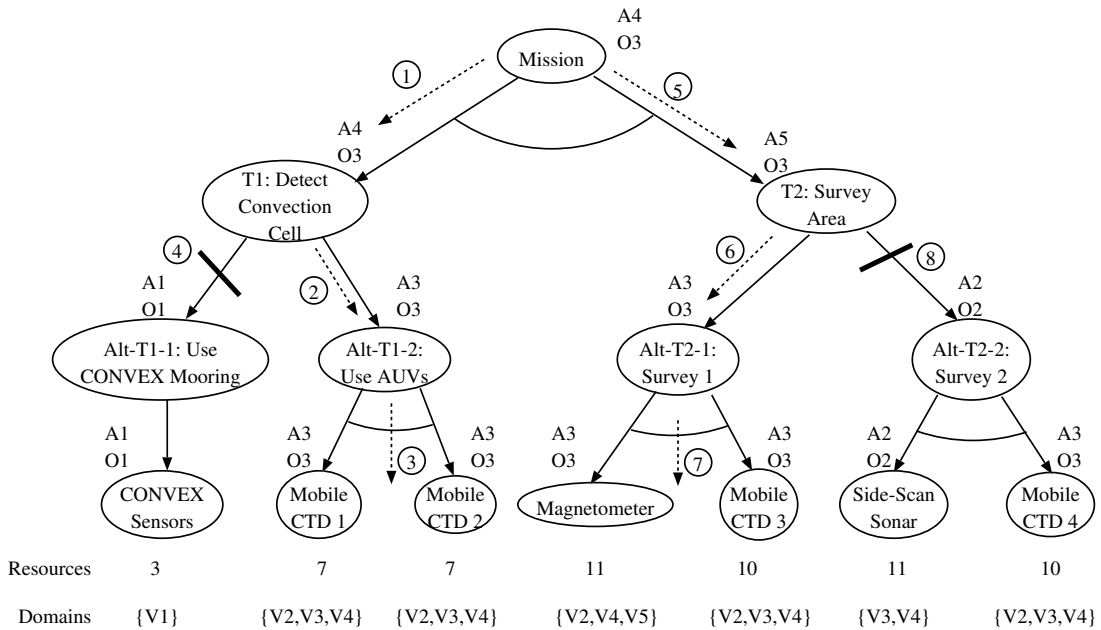

- State: TDT + constraint graph (initially empty)
- Operators: add to constraint graph, set value
- Perform CHS algorithm on constructed constraint graph