Constraint Satisfaction

UMaine COS 470/570 – Introduction to AI

Spring 2019

Search

Search so far...

- Uninformed search – nothing known about state space
- Heuristic search – *something* known, at least – defeasible
- Both: searching for a state with little internal structure
- Many problems: state has internal structure
- Important class of problems: state is assignment of *values* to *variables*

Artificial Intelligence

Examples

- Cryptarithmetic: Assign 0–9 uniquely to letters so that a symbolic expression is valid

```
 SEND
+MORE
-----
MONEY
```


Artificial Intelligence

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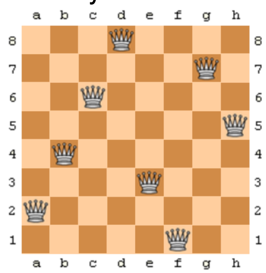
- N-queens problem: Place n queens on an $n \times n$ chessboard so that they don't attack one another



(From okpanico.files.wordpress.com)

Examples

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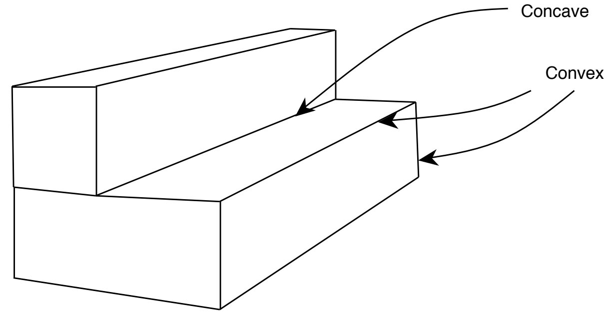
Special cases
Miscellaneous
Application: Task assignment in CoDA

- Computer vision: Classify edges in an image as convex or concave



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- Solving simultaneous equations

$$\begin{aligned} 3x + 4y + 6z &= 3 \\ 4x + 6y - 3z &= 4 \\ 7x - 3y - 4z &= 10 \end{aligned}$$

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- Solving simultaneous equations

$$\begin{aligned} 3x + 4y + 6z &= 3 \\ 4x + 6y - 3z &= 4 \\ 7x - 3y - 4z &= 10 \end{aligned}$$

- BSAT: Is a sum-of-products binary expression satisfiable, and if so, with what T/F assignments?

$$ABC + \overline{A}\,\overline{B}\,C + \cdots + ABC\overline{D}$$

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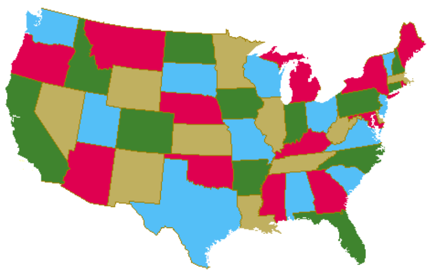
Examples

- Map coloring: Can we color a map of connected regions with n colors without two adjacent regions having the same color?



(From people.math.gatech.edu/~thomas)

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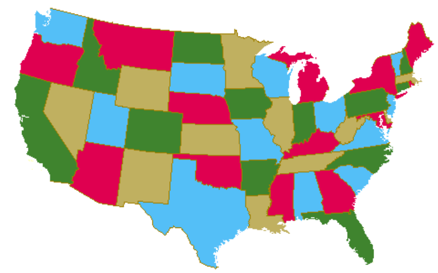

(From people.math.gatech.edu/~thomas)

- Scheduling: Scheduling a meeting with n people

Constraints

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  - Often very large search spaces:

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- Better approach:

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- Constraints can focus search: concentrate where variables constrain each other (e.g.)
- Sometimes: radically reduce search effort

Constraint satisfaction problems

Constraint-satisfaction problems

- *Constraint satisfaction problems* (CSPs):
  - Require set of variables to be bound to values $\in$ domain
  - Require constraints to be *satisfied*
- Instead of trying all possible variable/value assignments via search...
- *Propagate* constraints and values $\Rightarrow$ reduce domains of variables
- $\mathcal{O}(v^d)$ in w.c.: try $\Rightarrow \mathcal{O}(v^{d'})$, $d' \ll d$ in average case
- Fox, others: All problems can be reformulated $\Rightarrow$ CSPs

Constraints

- Types by arity:
  - *Unary constraints:* constraint on single value
  - *Binary, ternary, n-ary constraints:* restrict value of variable depending on value of other variable(s)
  - All $n - ary$ constraints can be $\Rightarrow$ binary constraints
- Types by whether absolute or preference constraints

Constraint propagation

- Suppose we reduce domain of v_1 that constrains v_2
- Some values in v_2 might now be eliminated
- Thus decision at v_1 *propagates* via the constraint to v_2
- Propagation continues from v_2, etc.
- May ultimately change v_1 again
- Stop when no more changes occur
- More constrained the values $\rightarrow$ faster to a solution

An Example of constraint propagation

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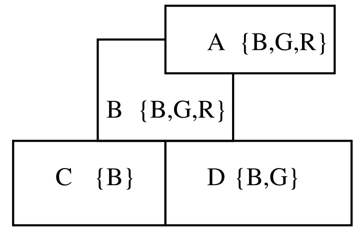

Map-coloring Problem

- Cannot color adjacent areas with the same color
- Some areas may have unary constraints which limit their domains
- Eliminate impossible assignments by propagating constraints

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CSP formalism

Constraint satisfaction problem

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- *Constraint graph*

Constraint satisfaction problem

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Constraint satisfaction problem

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- *Constraint graph*
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- One possible Constraint representation: *intensionally* – e.g., $v_1 \neq v_2$

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  - *Extensionally* – list values that satisfy constraint

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- Constraint $C = \{(d_1, d_2) \mid d_1 \in \mathrm{dom}(v_1)\ \&\ d_2 \in \mathrm{dom}(v_2)\}$

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- Constraint $C = \{(d_1, d_2) \mid d_1 \in \text{dom}(v_1) \,\&\, d_2 \in \text{dom}(v_2)\}$
- Goal: All variables instantiated, no violated constraints

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What is a state?

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- State representation 1: *complete assignments*
  - Start: Graph + random assignments
  - Operator: Change variable's value
  - Goal: All constraints satisfied
  - *Generate and test* search:
    - Set variable, check for goal
    - No guidance on which variable, value to choose
    - Quickly intractable: $n!d^n$ leaves

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    - Quickly intractable: $n!d^n$ leaves
    - E.g.: For 4-coloring of 48 states: ~10^{90}
- State representation 2: *partial assignments*
  - State: Graph + domains – singleton = assignment
  - Operator: Make assignment
  - After each assignment: propagate constraints
  - Goal: all singleton domains
  - Encounter empty domain: backtrack
  - Systematically explore space by choosing how vars instantiated

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Constraint Propagation

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Constraint propagation

- After value selected, propagate effects using constraints
- Propagation $\Rightarrow$ narrowing of domains to be consistent
- Two types of consistency:
  - Node consistency
  - Arc consistency

Node consistency

Constraint Satisfaction

- Check unary constraints
- Pre-processing, $\mathcal{O}(n)$ step
- E.g.:
  - Map-coloring problem: $\forall\, v\ \mathrm{dom}(v) = \{red, green, blue, yellow\}$
  - Texans object to blue, $\mathrm{dom}(\mathrm{Texas}) = \{red, green, yellow\}$
- All unary constraints satisfied: graph is *node-consistent*
- Unary constraints reduce |domain| $\Rightarrow$ prunes search tree, $\downarrow$ branching factor

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Arc Consistency

Constraint Satisfaction

- Eliminate any constraint violations

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Arc Consistency

Constraint Satisfaction

- Eliminate any constraint violations
- Pairwise checking of constraints, propagation of changes

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Arc Consistency

- Eliminate any constraint violations
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- Delete values from domain of variable if they are not consistent with all constraints on the variable

Arc Consistency

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- What does "consistent" mean?

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  - Value $y \in \text{dom}(v_2)$ is consistent with c iff $\exists x \in \text{dom}(v_1)\ \&\ (x, y) \in c$

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  - Special case of arc consistency

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  - Initiated when variable assigned value

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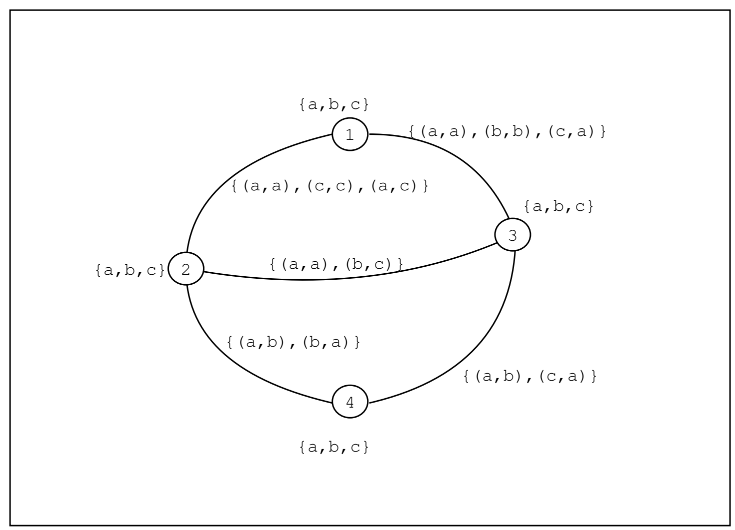
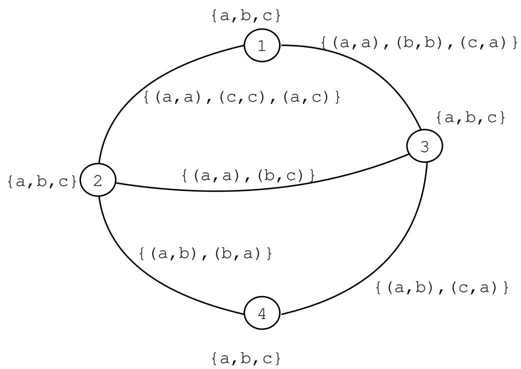


Example


```
Tuples listed so that lower-numbered
variable is to left.
```



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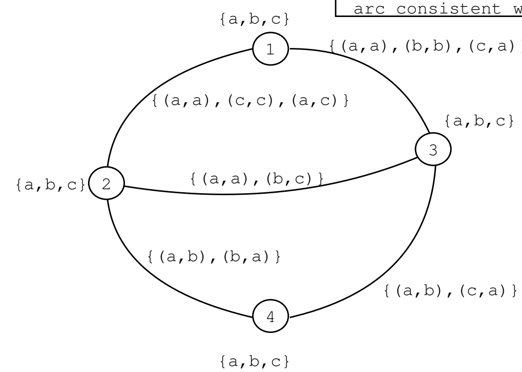
Example


```
Look at 1 & 3: 1 is
arc consistent with 3
```



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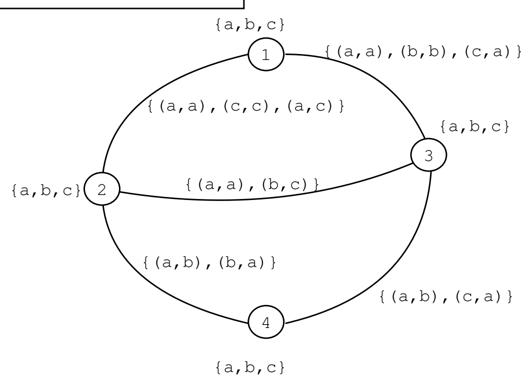
Example


```
Look at 1 & 2:
   No value in 2's domain
   consistent with 1=b.
```



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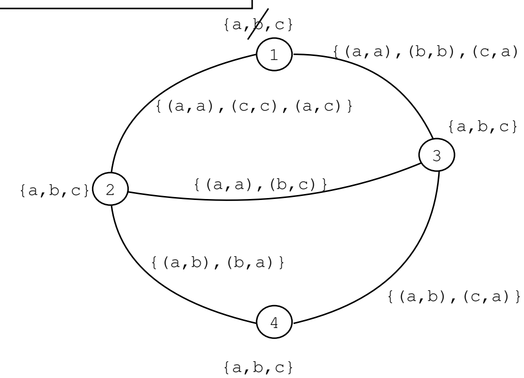

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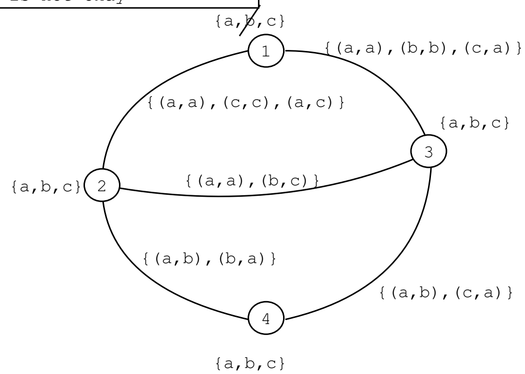
Example


```
Lookat 2 & 1:
  2=a is okay:  (a,a)
  2=c is okay: (c,c) and (a,c)
  2=b is not okay
```



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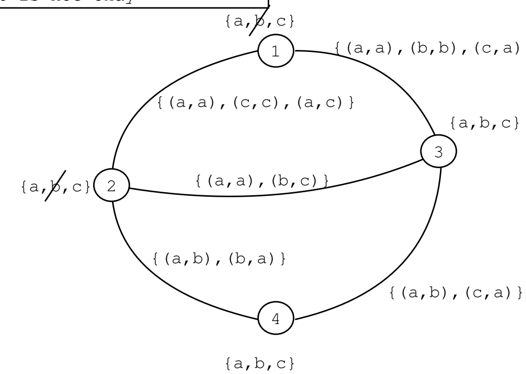

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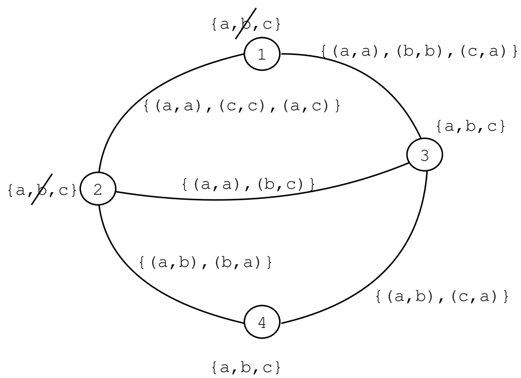

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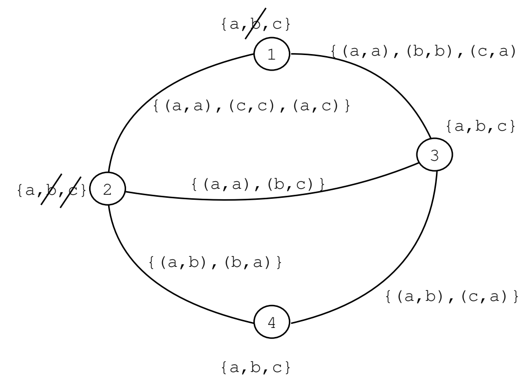

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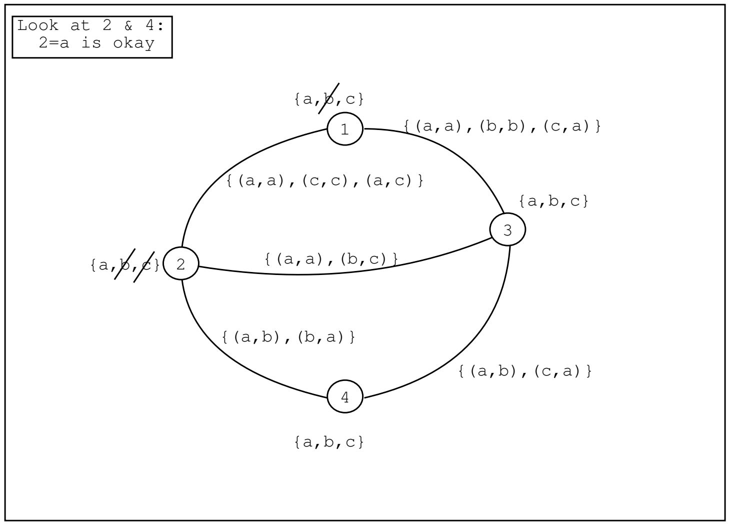

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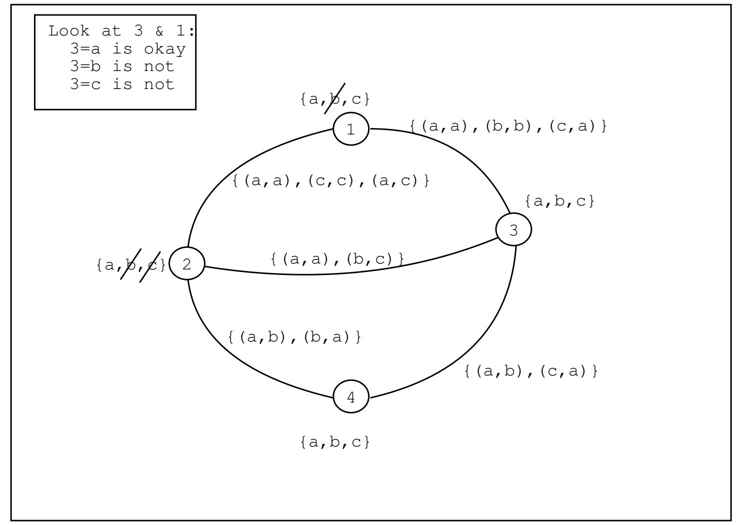

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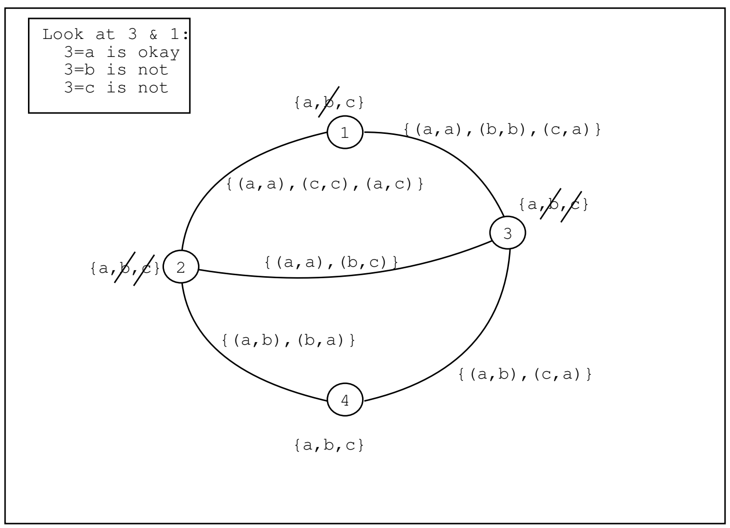

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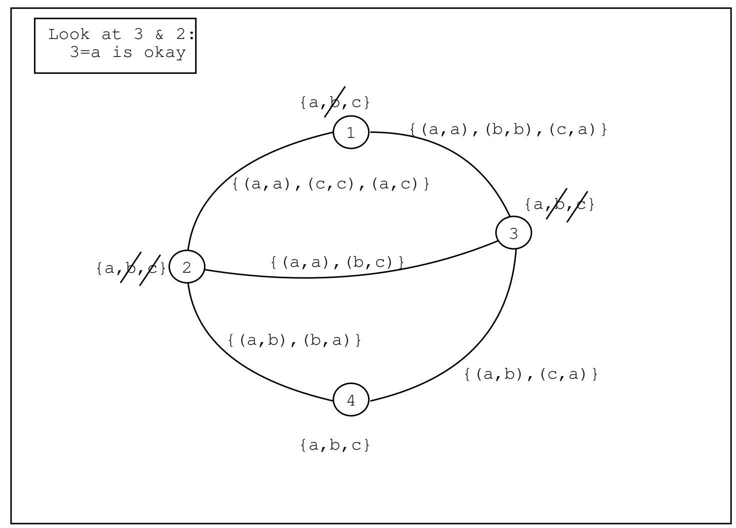

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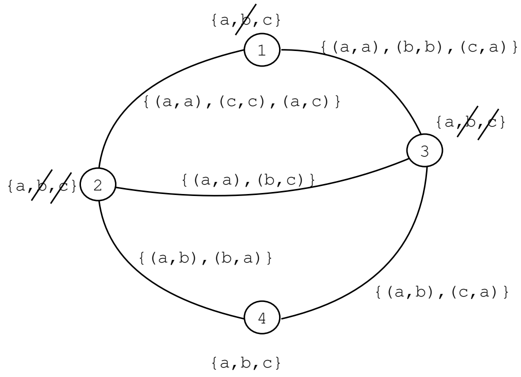
Example

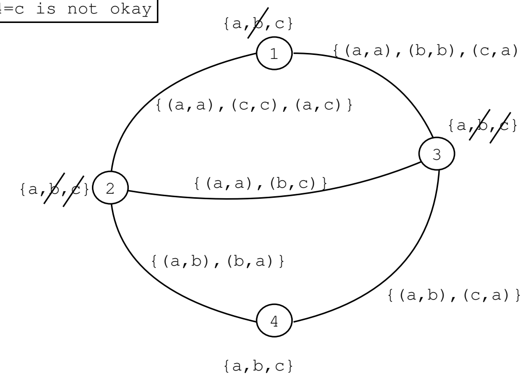
Look at 3 & 4:
3=a is okay



Artificial Intelligence

Example

Look at 4 & 2:
4=a is not okay
4=b is okay
4=c is not okay



Constraint Satisfaction

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Artificial Intelligence

Example

Look at 4 & 2:
4=a is not okay
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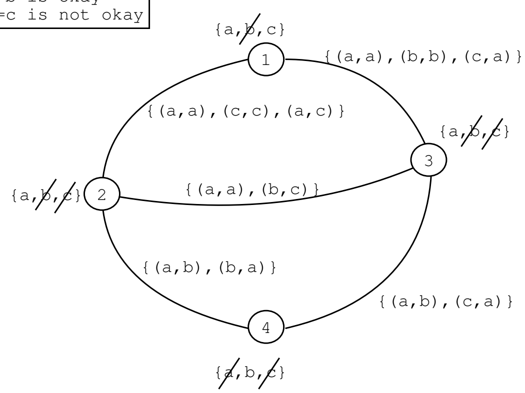

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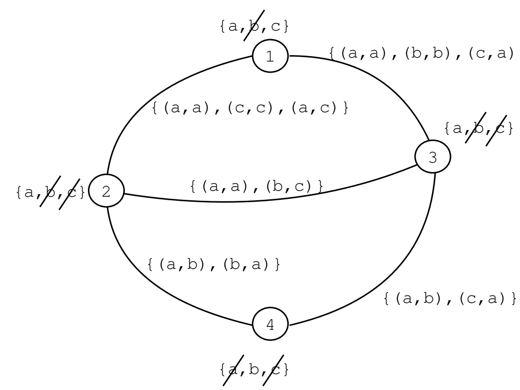
Example

Look at 4 & 3:
4=b is okay



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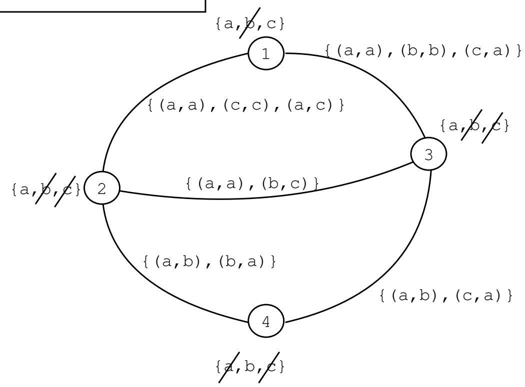
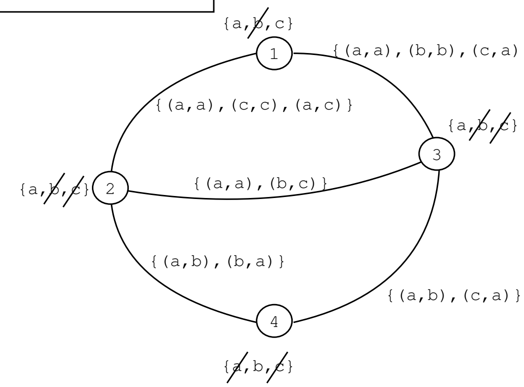
Example

```
At this point: Is CSP
arc-consistent?
```



Example

```
At this point: Is CSP
arc-consistent?
No!  1=c won't work!
```



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Have to continue
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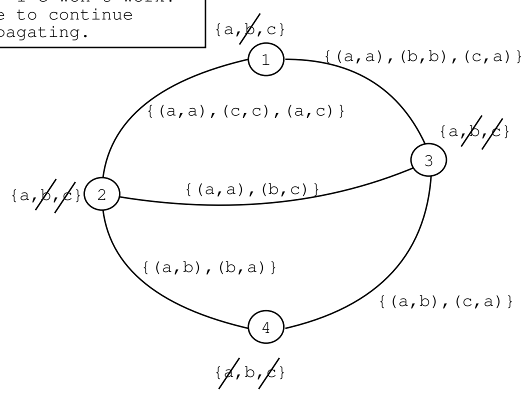

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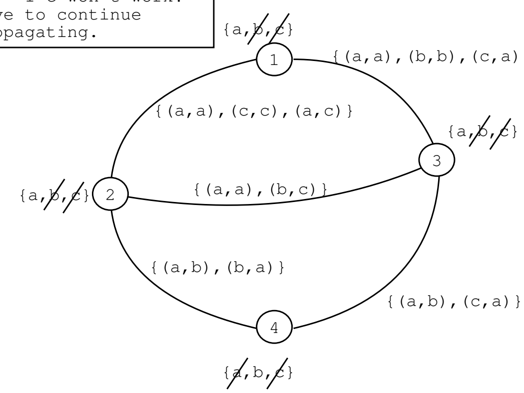

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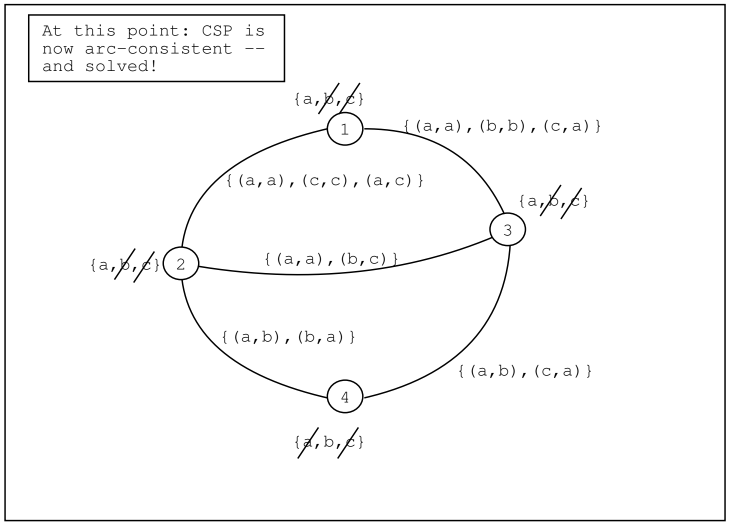

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Your turn

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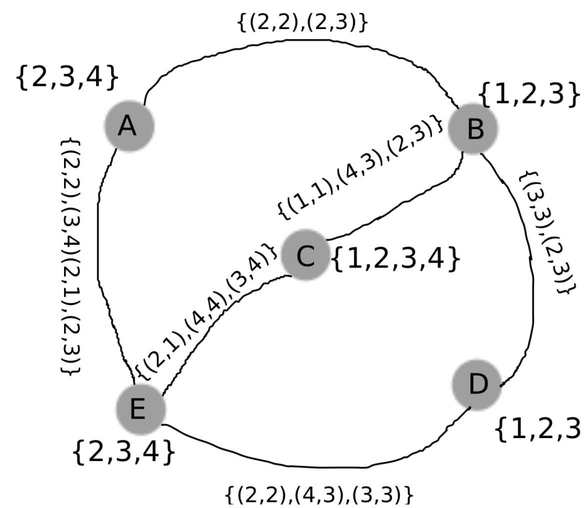

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Artificial Intelligence

Example of CSP

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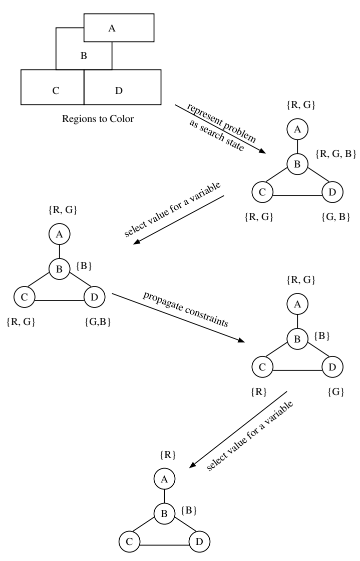

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CSP as search

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- Best case: value selection + propagation →→ solution

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CSP as search

- Best case: value selection + propagation →→ solution
- But it's a search process

CSP as search

- Best case: value selection + propagation →→ solution
- But it's a search process :
  - But what if dead-end?

CSP as search

- Best case: value selection + propagation →→ solution
- But it's a search process :
  - But what if dead-end?\ Backtrack

CSP as search

- Best case: value selection + propagation →→ solution
- But it's a search process :
  - But what if dead-end?\ Backtrack
  - And *which* variable, *which* value to pick each choice point?

Heuristics for CSP search

- Which variable to set?

Heuristics for CSP search

Constraint Satisfaction

- Which variable to set?
  - *Most-constrained variable heuristic:*

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Heuristics for CSP search

Constraint Satisfaction

- Which variable to set?
  - *Most-constrained variable heuristic:*
    - Pick variable with smallest remaining domain

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Heuristics for CSP search

Constraint Satisfaction

- Which variable to set?
  - *Most-constrained variable heuristic:*
    - Pick variable with smallest remaining domain
    - Reduces branching factor: fewest alternatives to backtrack to

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Heuristics for CSP search

- Which variable to set?
  - *Most-constrained variable heuristic:*
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  - *Most-constraining variable heuristic:*

Heuristics for CSP search

- Which variable to set?
  - *Most-constrained variable heuristic:*
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    - Reduces branching factor: fewest alternatives to backtrack to
  - *Most-constraining variable heuristic:*
    - Assign variable with most constraints

Heuristics for CSP search

- Which variable to set?
  - *Most-constrained variable heuristic:*
    - Pick variable with smallest remaining domain
    - Reduces branching factor: fewest alternatives to backtrack to
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Heuristics for CSP search

- Which variable to set?
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Heuristics for CSP search

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- Which value to use?
  - *Least-constraining value heuristic:*

Heuristics for CSP search

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- Which variable to set?
  - *Most-constrained variable heuristic:*
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    - Reduces branching factor: fewest alternatives to backtrack to
  - *Most-constraining variable heuristic:*
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- Which value to use?
  - *Least-constraining value heuristic:*
    - Choose value that rules out fewest values from connected variables

Heuristics for CSP search

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- Which variable to set?
  - *Most-constrained variable heuristic:*
    - Pick variable with smallest remaining domain
    - Reduces branching factor: fewest alternatives to backtrack to
  - *Most-constraining variable heuristic:*
    - Assign variable with most constraints
    - Reduces branching factor by pruning other variables' domains
- Which value to use?
  - *Least-constraining value heuristic:*
    - Choose value that rules out fewest values from connected variables
    - increases likelihood of success

Example

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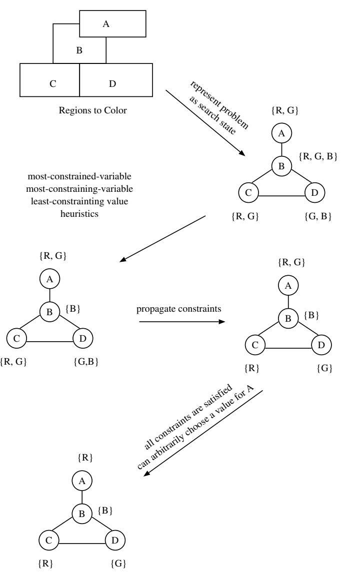

Special cases

Special cases

- *Independent subproblems:*

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  - Identify connected components of graph, solve separately

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  - Becomes *linear* in n: $\mathcal{O}(n/c \times d^c)$

Special cases

- *Independent subproblems:*
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  - Suppose each subproblem has c variables of total n
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  - $n = 80, \ d = 2, \ c = 20, 10^7$ nodes/sec: 4 billion years without, 0.4 s with

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- *Acyclic constraint graph:*

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  - Pick root, order nodes parent $\rightarrow$ child

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Special cases

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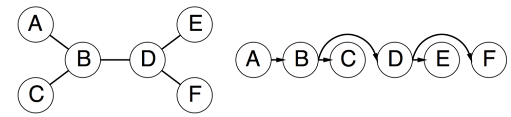

  - From leaves → root, remove inconsistencies between child, parent

(From S. Russell's slides)

Artificial Intelligence

Special cases

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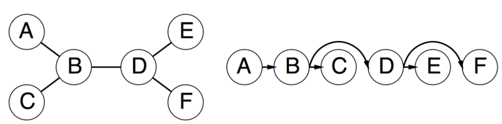

  - From leaves → root, remove inconsistencies between child, parent
  - From root → leaves: pick value consistent w/ parent

(From S. Russell's slides)

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Special cases

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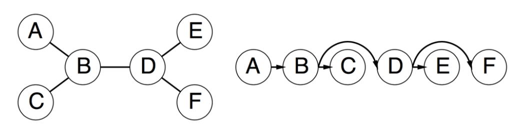

  - From leaves → root, remove inconsistencies between child, parent
  - From root → leaves: pick value consistent w/ parent
  - $\mathcal{O}(nd^2)$

(From S. Russell's slides)

Artificial Intelligence

Special cases

- *Almost tree-structured:*

(*From S. Russell's slides*)

Special cases

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Application: Task assignment in CoDA

- *Almost tree-structured:*
  - Instantiate set of variables in all possible ways s.t. remainder is tree-structured

(*From S. Russell's slides*)

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- *Almost tree-structured:*
  - Instantiate set of variables in all possible ways s.t. remainder is tree-structured
  - Take out the *cutset*

(*From S. Russell's slides*)

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- *Almost tree-structured:*
  - Instantiate set of variables in all possible ways s.t. remainder is tree-structured
  - Take out the *cutset*
  - If cutset size c, $\mathcal{O}(d^c \cdot (n-c)d^2)$

(*From S. Russell's slides*)

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Artificial Intelligence

Iterative algorithms

- Use hill-climbing, simulated annealing

(From S. Russell's slides)

Iterative algorithms

Constraint Satisfaction

- Use hill-climbing, simulated annealing
- Complete assignment, allow violated constraints

(From S. Russell's slides)

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Iterative algorithms

Constraint Satisfaction

- Use hill-climbing, simulated annealing
- Complete assignment, allow violated constraints
- Operators: reassign variables

(From S. Russell's slides)

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Iterative algorithms

Constraint Satisfaction

- Use hill-climbing, simulated annealing
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(From S. Russell's slides)

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Iterative algorithms

- Use hill-climbing, simulated annealing
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- Operators: reassign variables
- Select any variable
- Value: use *min-conflicts* heuristic – choose state w/ fewest constraints violated

(From S. Russell's slides)

Iterative algorithms

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- How good?

(From S. Russell's slides)

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Iterative algorithms

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  - Result for (e.g.) n-queens

(From S. Russell's slides)

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- How good?
  - Result for (e.g.) n-queens
  - Can solve in almost $\mathcal{O}(n)$ time with high probability

(From S. Russell's slides)

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Iterative algorithms

- Use hill-climbing, simulated annealing
- Complete assignment, allow violated constraints
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- Select any variable
- Value: use *min-conflicts* heuristic – choose state w/ fewest constraints violated
- How good?
  - Result for (e.g.) *n*-queens
  - Can solve in almost $\mathcal{O}(n)$ time with high probability
  - For almost any number of queens

(From S. Russell's slides)

Miscellaneous

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Continuous variables

- Many real-world problems – e.g., scheduling times for space applications, etc.
- If linear constraints: solvable by *linear programming* in polynomial time

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Constrained Heuristic Search (CHS)

- Can we use even more heuristic information?
- CHS (Fox et al., 1989): Constraint graphs become states in state space search graph
- Operators: assign value, add/delete constraint, constrain domain of variable
- Heuristics: look for *textures* in graph $\Rightarrow$ operator to apply

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Application: Task assignment in CoDA

CoDA

- Autonomous oceanographic sampling networks (AOSNs)

CoDA

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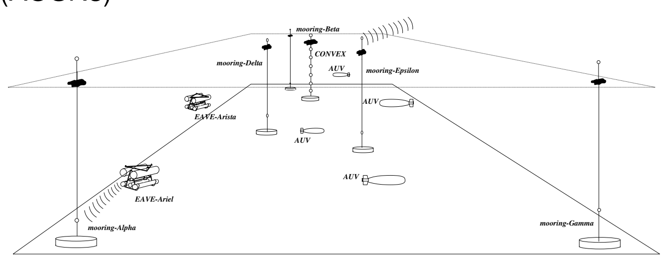
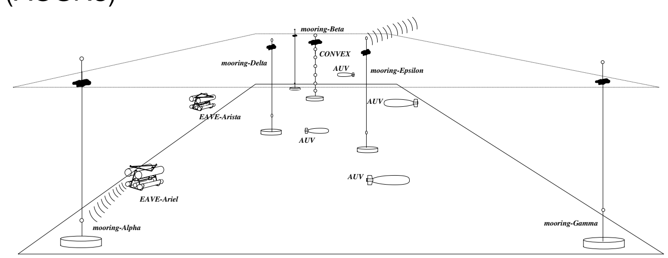


CoDA

- Autonomous oceanographic sampling networks (AOSNs)



- Treat as multiagent systems (MAS): CoDA (Turner & Turner)
- Need task assignment: Constraint satisfaction problem
- Use CHS

Overview

- Identify *capabilities*: of AUVs, needed for problem
- Create *task-decomposition tree*

Artificial Intelligence

Overview

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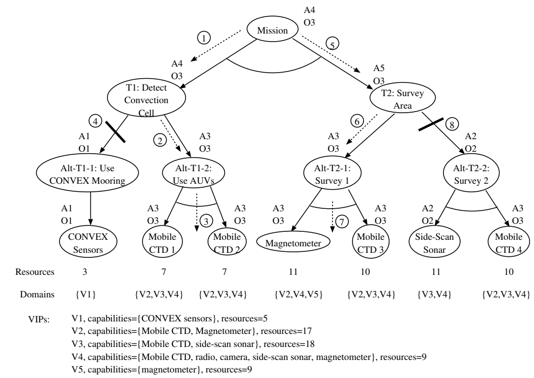

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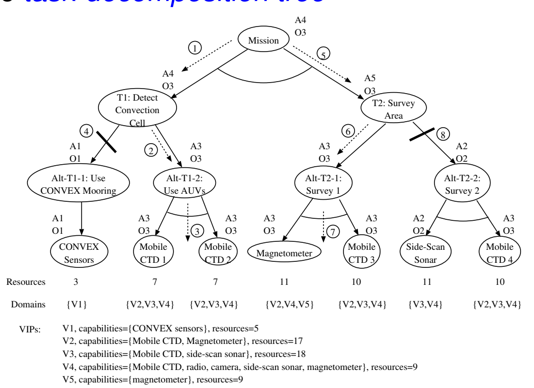

- State: TDT + constraint graph (initially empty)
- Operators: add to constraint graph, set value
- Perform CHS algorithm on constructed constraint graph

Artificial Intelligence