COS 140: Foundations of Computer Science

Designing Circuits from Specifications

Fall 2018

Homework

- Reading: Chapter 6
- Homework:
  - All exercises at the end of Chapter 6
  - Due Monday, 9/24
  - All homework is due in class in *hardcopy* form on the due date!
- Cybersecurity team: See *umcst.maine.edu* for a description and email link.

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Circuit Specifications

Circuits

- Computer components realize *functions*
- Components are *logic circuits*
- How to describe a function – a *specification*
- Once there is a specification – how can we design a circuit for it?

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Circuit Specifications

- Two types: Boolean algebra and truth tables
- Boolean algebra expressions
  - Good when can state function in words, with logical operators

    E.g.: Create an expression that equals 1 when all of the inputs are 1, or when one, and only one, input is 0.
- Truth tables
  - Particularly good when relationship of input to output does not follow an easily-stated pattern.

    E.g.: Cards from a deck are assigned an input pattern arbitrarily.
  - Also good when you do not want to specify outputs for all inputs.

Example

Suppose our company makes jelly beans – 50 varieties of them total, but only some are made each month. How can we design a circuit that tells us, for any particular variety we are interested in, whether it is being made (1) or not being made (0) this month?

- Naïve way: 50 input lines, one input line for each variety
- Too many lines!

Example

50 varieties, not all made (1) each month.

- □ Better way: *code* the inputs
- □ Let each variety be a unique pattern of 1s in the input
- □ n input lines can represent 2^n different patterns
- □ How many input lines do we need?

n:	1	2	3	4	5	6
2^n:	2	4	8	16	32	64

- □ What about the leftovers?

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Example

50 varieties, not all made (1) each month.

- □ There are $64 - 50 = 14$ unused combinations
- □ We don't care whether their corresponding outputs are 1 or 0: *don't cares*
- □ We can leave these out of the truth table or mark with a special symbol (e.g., "–")

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Example

50 varieties, not all made (1) each month.

A	B	C	D	E	F	Being made?
...						
1	1	0	0	0	0	1
1	1	0	0	0	1	0
1	1	0	0	1	0	–
1	1	0	0	1	1	–
...						

With algebraic specification: have to decide on a value for each.

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Truth Tables with Don't Cares

- The circuit designer can choose the output value which is easiest to realize in the circuit for don't cares.
  - The circuit will have to have a value for each set of inputs.
  - Don't cares mean we "don't care" what that value is for some set of values for inputs variables.

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Creating Circuits from Algebraic Expressions

- Gates correspond to operators.
- Show input variables.
- Lines (representing physical connections) go from variables or output of a gate to input for a gate or output for the circuit.
- Gates should be visited in the order that the subexpressions are evaluated, as values "travel" along lines from input to output.
- Usually show circuits with input on left and output on right.

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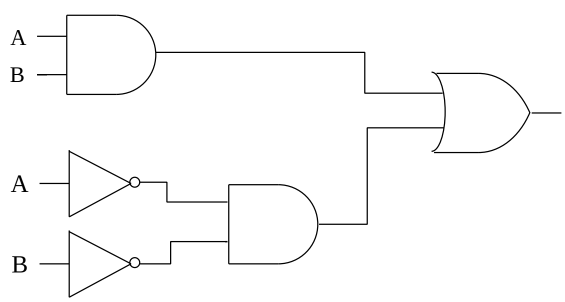
Example: Creating a Simple Circuit

Create a circuit with output 1 if and only if the value of both of its input variables are 1 or both are 0.

Algebraic specification: $AB + \overline{A}\,\overline{B}$



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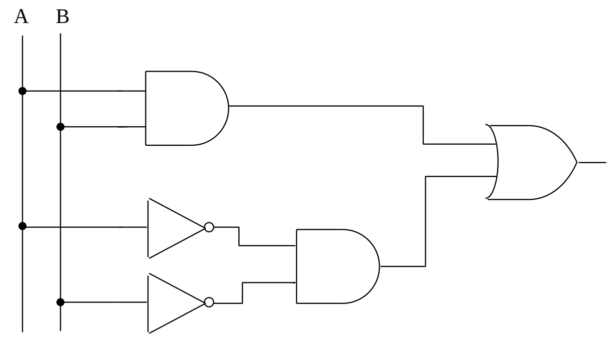
Example: Other Ways to Draw Circuits



Input values can be shown once, connected to input for appropriate gates.

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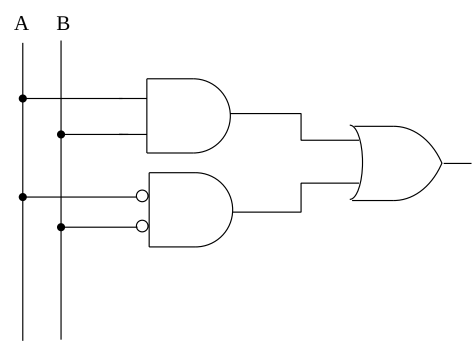
Example: Other Ways to Draw Circuits (cont'd)



NOT can be represented by a circle anywhere on the input line.

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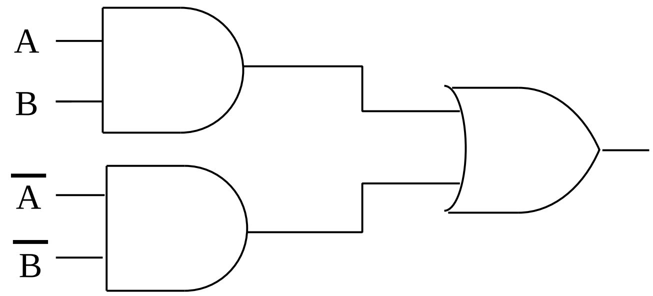
Example: Other Ways to Draw Circuits (cont'd)



Input values can be shown as negated.

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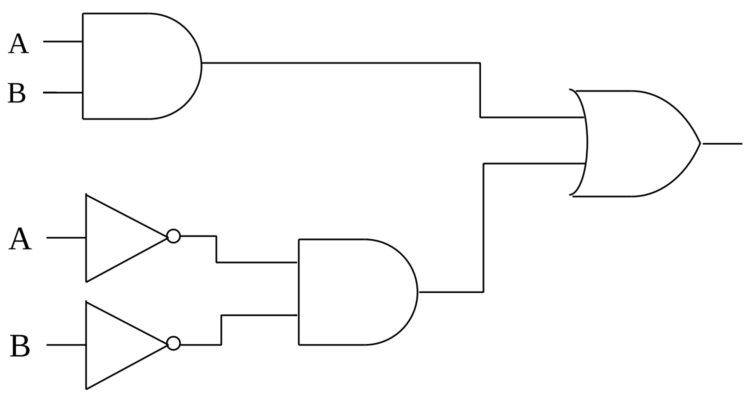
Example (cont'd): Following the Values



(Fill in values during lecture)

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Creating Circuits from Truth Tables

- Have truth table with inputs and values for sets of inputs given.

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

  - For each set of inputs (i.e., each row) with an output of 1, create a *product term*.
  - The product term contains each input variable (or its negation).
  - If the value of the input variable is 1, it appears in the product term. If the value of the input variable is 0, its negation appears in the product term.

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Creating Circuits from Truth Tables (cont'd)

A	B	C	F	Product Term
0	0	0	0	
0	0	1	0	
0	1	0	1	$\overline{A}B\overline{C}$
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$AB\overline{C}$
1	1	1	1	ABC

$$\Rightarrow \overline{A}B\overline{C} + AB\overline{C} + ABC$$

Sum of products (SOP) form.

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Creating Circuits from Truth Tables (cont'd)

- For these inputs, if each variable has the correct value, the output should be 1. AND ensures that all variables have the correct input value.
- OR together the product terms that were created in the previous step, so that *any* set of correct inputs produces a 1 as output.

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Creating Circuits from Truth Tables (cont'd)

- Once an SOP expression is formed, algebraic substitution or any method for creating equivalent expressions, can be used to create an equivalent expression that will be better for realizing in a circuit.
- A circuit is created from the resulting logical expression.

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Programmable Logic Arrays

What is a PLA?

- Programmable Logic Array (PLA) is a chip designed with NOT, AND, and OR gates so it can handle arbitrary SOP expressions.
- A subarray of ANDs creates product terms from the inputs.
  - All inputs and their negations are available to all AND gates.
- A subarray of ORs takes input from the ANDs and creates outputs.
  - The chip can calculate several functions at once because of the different outputs.
- Large chips on the order of 25 inputs and 15 outputs.

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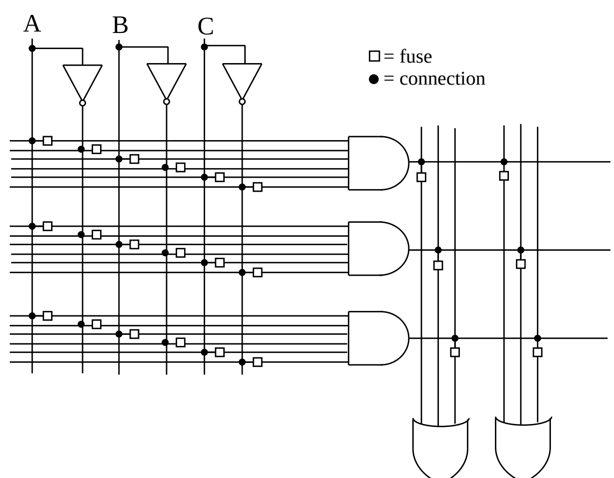
Example: A PLA



A small PLA with 3 inputs and 2 outputs.

This is a small PLA for illustrative purposes only. Can have any number of AND and OR gates.

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Using PLAs for SOPs

Advantages of a PLA

- Can capture all SOPs (within constraints of input size), and SOPs can be used to express any Boolean function.
- Several outputs
  - Can have one function for each output through an OR gate.
  - Can use calculation of a product in more than one function.
- Take advantage of the efficiency (cost, size, speed) of large-scale integration in a general-purpose way.

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Programming the PLA

Programming the PLA

- Field-programmable logic array
  - All connections are made using fuses at the intersection points.
  - Fuses are blown when the connection is not wanted in the circuit.
- Or PLA can be programmed during chip fabrication

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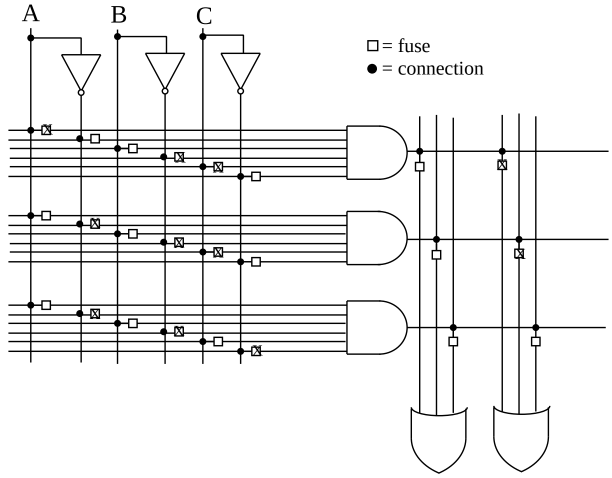
Example

A simple PLA programmed for $\overline{A}B\overline{C} + AB\overline{C} + ABC$ and ABC.



Fuses marked X will have been blown and will *not* be part of the input to the next gate.

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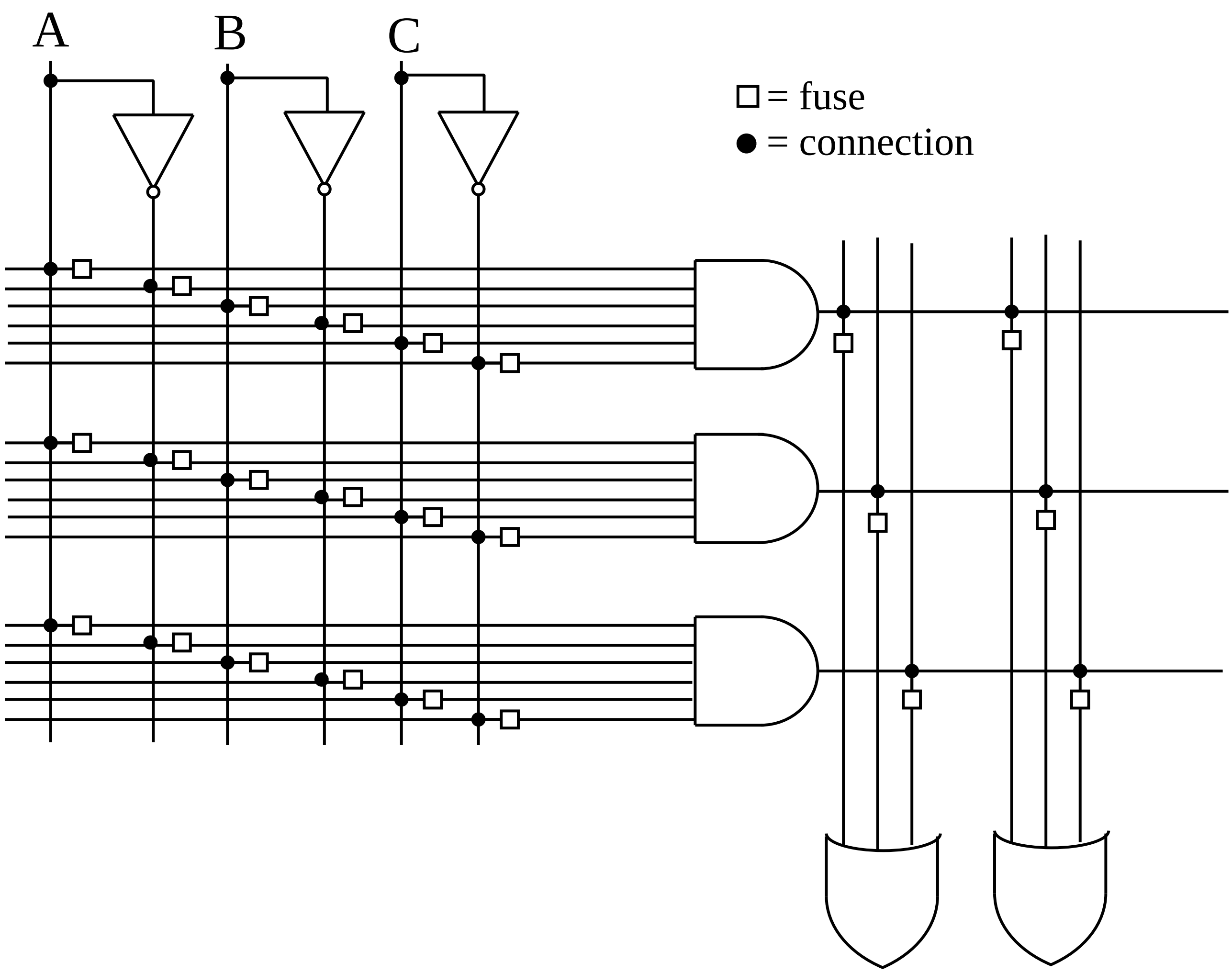
Example



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