Homework

- Reading: Chapter 8
- Exercises: Chapter 8, all
- Due Friday, 9/28

Computer Science Foundations

COS 140: Foundations of Computer Science

Adders

Fall 2018

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What is an adder?

- An adder is a logic circuit that adds binary numbers
- Could add two 1-digit numbers or two n-bit numbers

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Why Study Adders?

- Interesting example of a combinational circuit.
  - Circuit whose output relies solely on its inputs.
  - Perform an important function for the computer.
    - Addition is also basis for other arithmetic functions in the computer (subtraction, multiplication, etc.)
    - Would like the function done in hardware so it is done quickly.

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How Do We Do Addition?

1. Write down numbers that will be added using symbols from 0 to 9.
2. Use arithmetic facts to add numbers in a column. If more than 9, carry the most significant digit to the next column.

How Do We Do Addition?

1. Write down numbers that will be added using symbols from 0 to 9.
2. Use arithmetic facts to add numbers in a column. If more than 9, carry the most significant digit to the next column.

$$
\begin{array}{cccc}
 & & {\scriptstyle 1} & \\
 & 3 & 5 & 6 \\
+ & 2 & 3 & 5 \\
\hline
 & 5 & 9 & 1
\end{array}
$$

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Numbers and Digital Logic

- Symbols will correspond to the 0 or 1 that is the input or output of the circuit. So, have 2 symbols to work with, not 10.
- Create a binary system that is like our digital system.

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A Closer Look at Our Digital System

- Have 10 digits: 0–9
- Have “places” for 1’s, 10’s, 100’s, 1000’s, 10,000’s, etc. that correspond to powers of 10.

A Closer Look at Our Digital System

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  - $10^0 = 1; 10^1 = 10; 10^2 = 100; 10^3 = 1000; 10^4 = 10,000$
- To find the value of a number, add all the digits times their place values.

Computer Science Foundations

A Closer Look at Our Digital System

- Have 10 digits: 0–9
- Have “places” for 1’s, 10’s, 100’s, 1000’s, 10,000’s, etc. that correspond to powers of 10.
  - $10^0 = 1; 10^1 = 10; 10^2 = 100; 10^3 = 1000; 10^4 = 10,000$
- To find the value of a number, add all the digits times their place values.
  - $359 = 9 \times 1 + 5 \times 10 + 3 \times 100$

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In a Binary System...

- Have 2 digits: 0 and 1
- Places correspond to powers of 2

Computer Science Foundations

In a Binary System...

- Have 2 digits: 0 and 1
- Places correspond to powers of 2:

2^0	1	2^4	16	2^8	256
2^1	2	2^5	32	2^9	512
2^2	4	2^6	64	2^{10}	1024
2^3	8	2^7	128	2^{11}	2048

Computer Science Foundations

In a Binary System...

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2^1	2	2^5	32	2^9	512
2^2	4	2^6	64	2^{10}	1024
2^3	8	2^7	128	2^{11}	2048

- To find the value, add all the 1's and 0's times their place values.
  - $10110 = 0 \times 1 + 1 \times 2 + 1 \times 4 + 0 \times 8 + 1 \times 16 = 22$

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From decimal to binary

Given: n, a decimal number

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3. Decrement i to work on next-lower binary digit; if $i = 0$, we're done

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4. If $2^i > n$, then there should be a 0 for that power of two; write that down, and go to 3

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5. Else, if $2^i = n$, then write 0s for all the rest of the digits, and you're done

Computer Science Foundations

From decimal to binary

Given: n, a decimal number

1. First find the largest power of two less than n; let i be the exponent
2. Write down a 1, and $n = n$ minus that power of two
3. Decrement i to work on next-lower binary digit; if $i = 0$, we're done
4. If $2^i > n$, then there should be a 0 for that power of two; write that down, and go to 3
5. Else, if $2^i = n$, then write 0s for all the rest of the digits, and you're done
6. Otherwise ($2^i < n$), write a 1, since this power of 2 "fits" in n; $n = n - 2^i$, and go to 3

The algorithm

Algorithm Convert(d)
    Input: d, a decimal number
    Output: the binary version of d
    Let n be largest whole number such that $2^n \leq d$
    while $n \geq 0$ **do**
        if $d = 2^n$ **then**
            Output 1 followed by $n - 1$ 0s
            return
        else if $d < 2^n$ **then**
            Output 0
            $n = n - 1$
        else
            Output 1
            $d = d - 2^n$
            $n = n - 1$
        end if
    end while
End.

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Example

359

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Example

359 $\quad 2^8 < d < 2^9, \therefore n = 8 \Rightarrow$ 1

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Example

	359	$2^8 < d < 2^9, \therefore n = 8 \Rightarrow$ 1
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	359	$2^8 < d < 2^9, \therefore n = 8 \Rightarrow$	1
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	103	$2^7 > d;\ n = 6 \Rightarrow$	0
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	39		

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	39	$2^5 < d \Rightarrow$	1
-	32	Subtract $2^5 = 32,\ n = 4$	
	7		

Example

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	7	$2^4 > d; \; n = 3 \Rightarrow$	0
		$2^3 > d; \; n = 2 \Rightarrow$	0

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Adders

Binary Numbers

- A Closer Look at Our Digital System
- In a Binary System...
- From decimal to binary
- Algorithm

Adders

Multi-bit Adders

Example

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- 32	Subtract $2^5 = 32,\ n = 4$	
7	$2^4 > d;\ n = 3 \Rightarrow$	0
	$2^3 > d;\ n = 2 \Rightarrow$	0
	$2^2 < d \Rightarrow$	1
- 4	Subtract $2^2 = 4,\ n = 1$	
3		

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Example

359	$2^8 < d < 2^9, \therefore n = 8 \Rightarrow$	1
- 256	Subtract $2^8 = 256,\ n = 7$	
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7	$2^4 > d;\ n = 3 \Rightarrow$	0
	$2^3 > d;\ n = 2 \Rightarrow$	0
	$2^2 < d \Rightarrow$	1
- 4	Subtract $2^2 = 4,\ n = 1$	
3	$2^1 < d \Rightarrow$	1
- 2	Subtract $2^1 = 2,\ n = 0$	
1		

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Example

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	$2^2 < d \Rightarrow$	1
- 4	Subtract $2^2 = 4, \; n = 1$	
3	$2^1 < d \Rightarrow$	1
- 2	Subtract $2^1 = 2, \; n = 0$	
1	$2^0 = d, \Rightarrow$	1

Example

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-	4	Subtract $2^2 = 4,\ n = 1$	
	3	$2^1 < d \Rightarrow$	1
-	2	Subtract $2^1 = 2,\ n = 0$	
	1	$2^0 = d, \Rightarrow$	1

So $359_{10} = 101100111_2$

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There are only 10 kinds of people in this world. Those that understand binary and those that don't.

– graduate student's T-shirt

Use Arithmetic Facts to Add Numbers

- Addition results from applying facts about arithmetic to numbers
- For the computer to use arithmetic facts, we need to construct a circuit.
- So: start with a truth table.
- Construct a truth table for all of the inputs, including the possible carry.

Truth Table for Addition

Carry-in	A	B	Carry-out	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

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Circuit from truth table

- Can we find a circuit for this? A minimal circuit?

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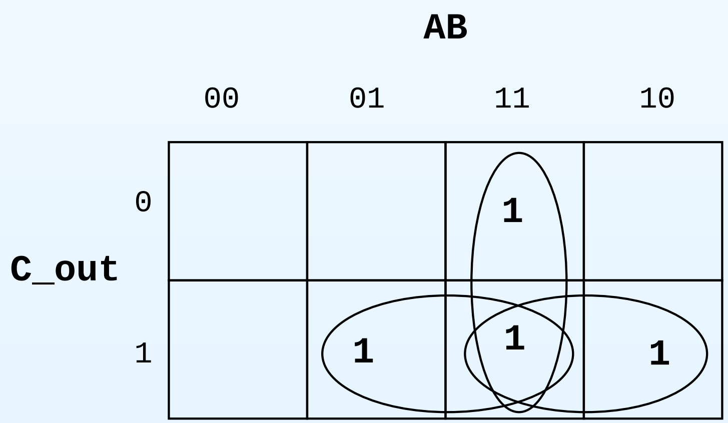
Circuit from truth table

- Can we find a circuit for this? A minimal circuit?
- Karnaugh map for carry out:

C_in	A	B	C_out	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



AB + BC + AC

Circuit from truth table

- Can we find a circuit for this? A minimal circuit?
- Karnaugh map for sum out:

C_in	A	B	C_out	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

AB

Sum	00	01	11	10
0		**1**		**1**
1	**1**		**1**	

~A~BC + ~AB~C +
ABC + A~BC

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A better idea

- So minimization using Karnaugh maps, algebraic substitution – not so good!
- Can we do better?

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Computer Science Foundations

A better idea

- So minimization using Karnaugh maps, algebraic substitution – not so good!
- Can we do better?
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- Things are simplified when we look at just A and B as inputs:

A	B	Carry	Sum
0	0	0	0
0	1	0	1
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- Sum and carry – both correspond to basic operations/gates

Computer Science Foundations

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- Sum = $A \oplus B$

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- Sum and carry – both correspond to basic operations/gates
- Sum = $A \oplus B$
- Carry = AB

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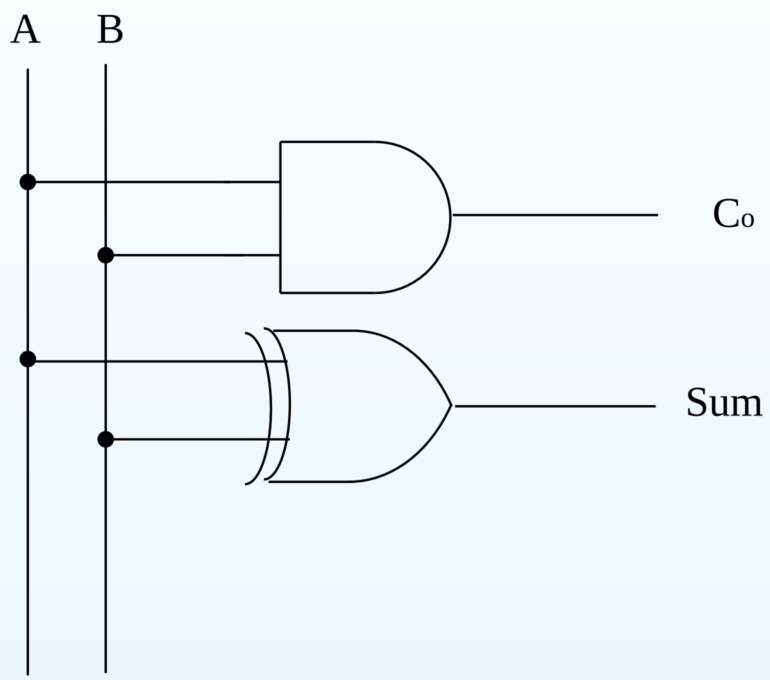
Half-Adder

We can create a very simple circuit to add A and B.



Half-adder because only does half the job.

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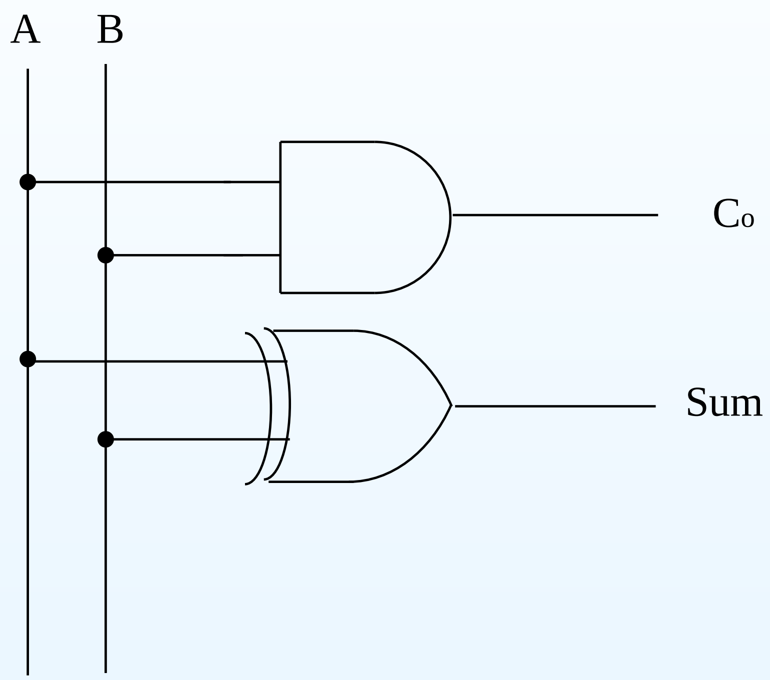

Half-adder because only does half the job.

We need a *full adder* that adds $A + B + C_{in}$

Full Adder

- A, B, $C_{in} \longrightarrow S$ (sum), C (carry out)
- Can we use a half-adder + additional logic get outputs?

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Full Adder

- $A, B, C_{in} \longrightarrow S$ (sum), C (carry out)
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Computer Science Foundations

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  - $S = A + B + C_{in}$

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  - Use another half-adder: $S_h, C_{in} \longrightarrow S_{h2} = S$

Full Adder

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Full Adder

- When is C (carry out) $= 1$?

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Full Adder

- When is C (carry out) $= 1$? When $A + B + C_{in} \geq 10_2$

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- When is C (carry out) $= 1$? When $A + B + C_{in} \geq 10_2$
  - Case 1: $A + B = 10_2$

Computer Science Foundations

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- When is C (carry out) $= 1$? When $A + B + C_{in} \geq 10_2$
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Computer Science Foundations

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  - Case 2: $A + B = 1$ and $C_{in} = 1$
    - This means that $S_h = 1$, $C_{in} = 1$
    - In this case, carry out of second half-adder $C_{h2} = 1$

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  - So $C = 1$ when either either or both half-adder carries is 1
  - $\therefore C = C_h \vee C_{h2}$

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Full Adder

We can create a full adder by putting two half adders together as described above.

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Full Adder

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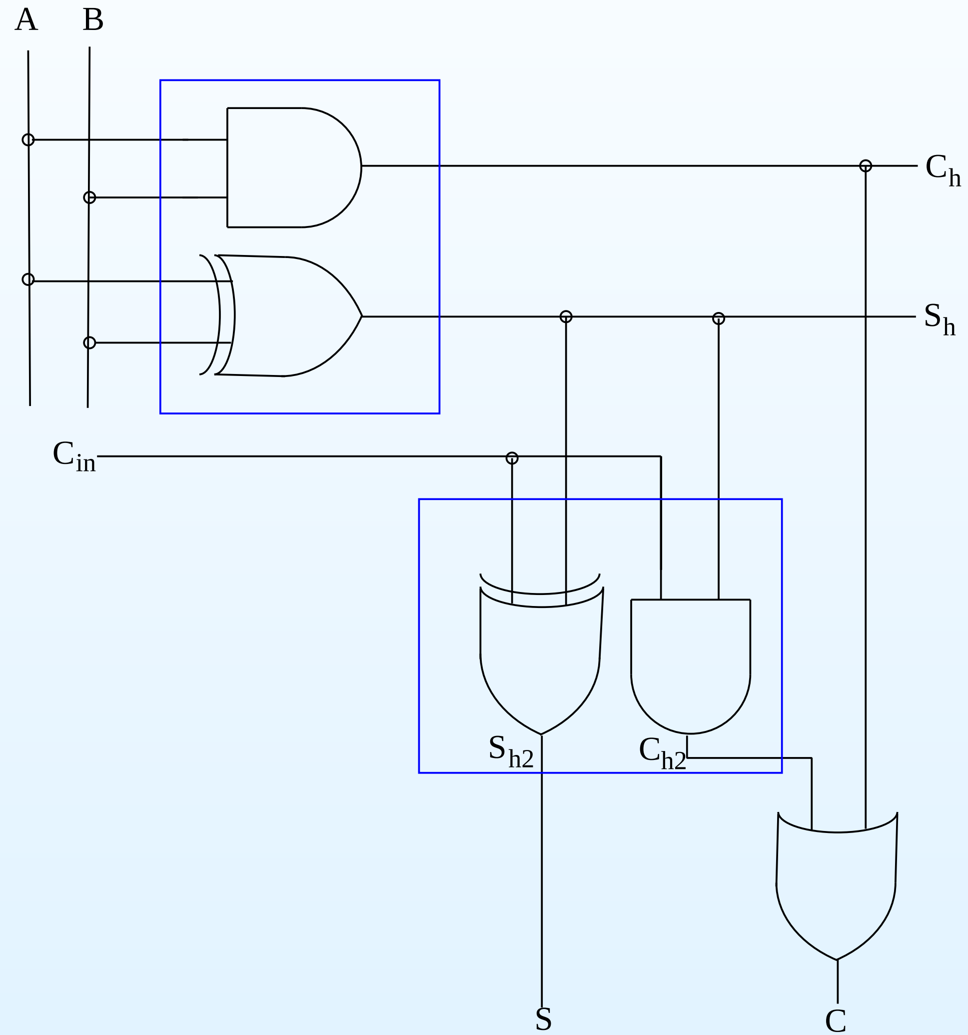
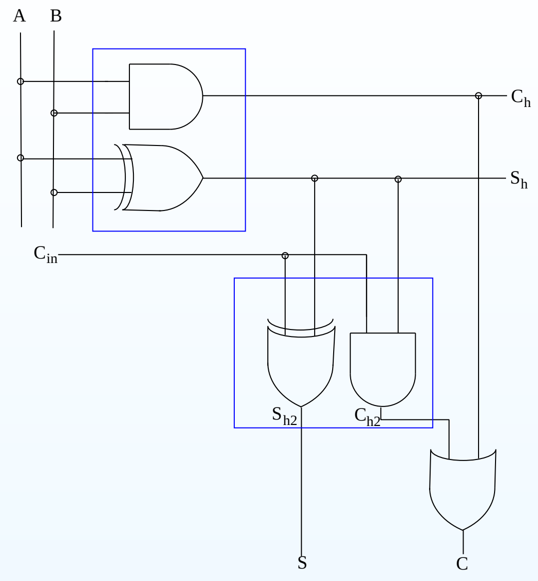


Full Adder



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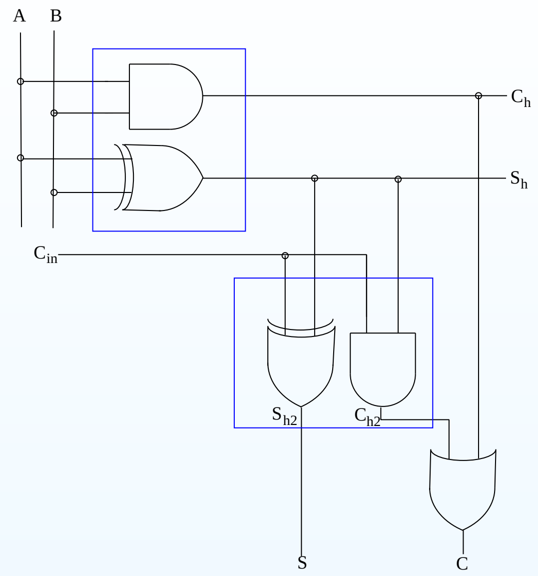
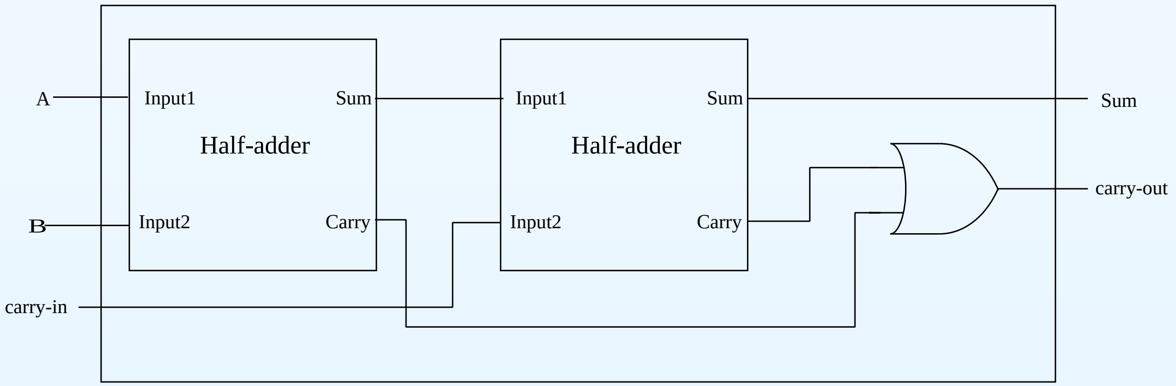
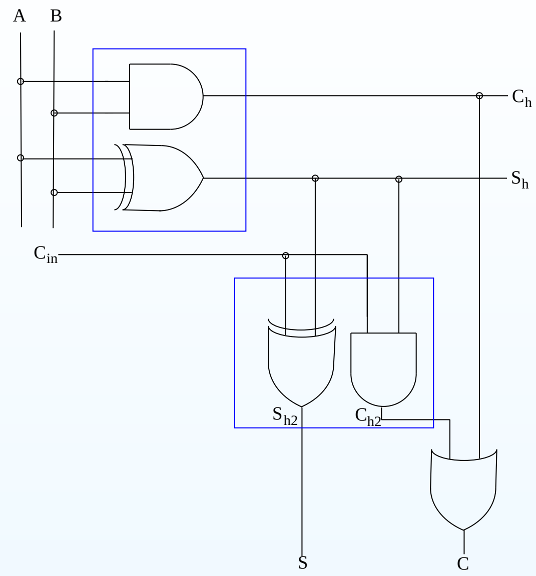
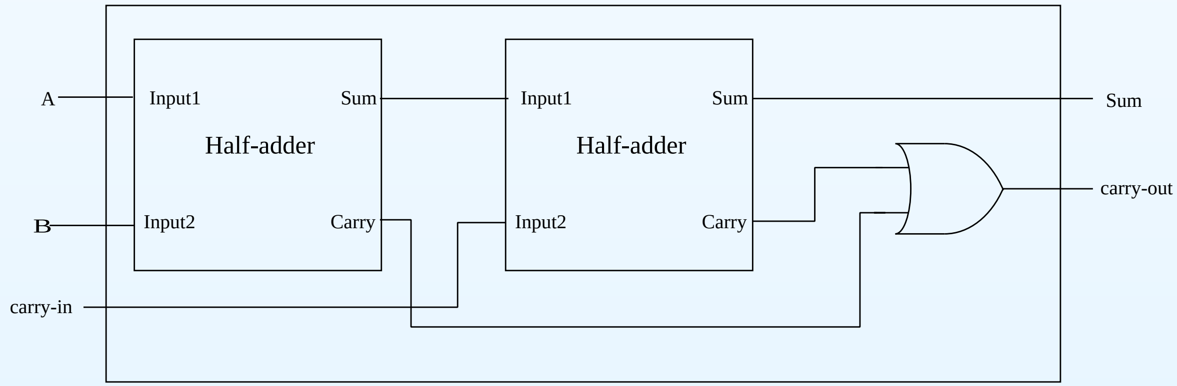
Full Adder

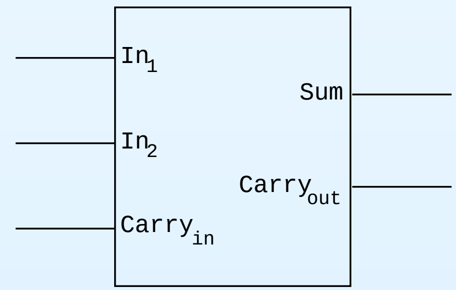

A
B
C_h
S_h
C_{in}
S_{h2}
C_{h2}
S
C


A
Input1
Sum
Half-adder
B
Input2
Carry
Input1
Sum
Half-adder
Input2
Carry
Sum
carry-out
carry-in

Full Adder







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Creating Multi-Bit Adders

- Add multi-digit binary numbers using a full-adder for each bit.
- Problem: How to compute the carry-in for adder n?

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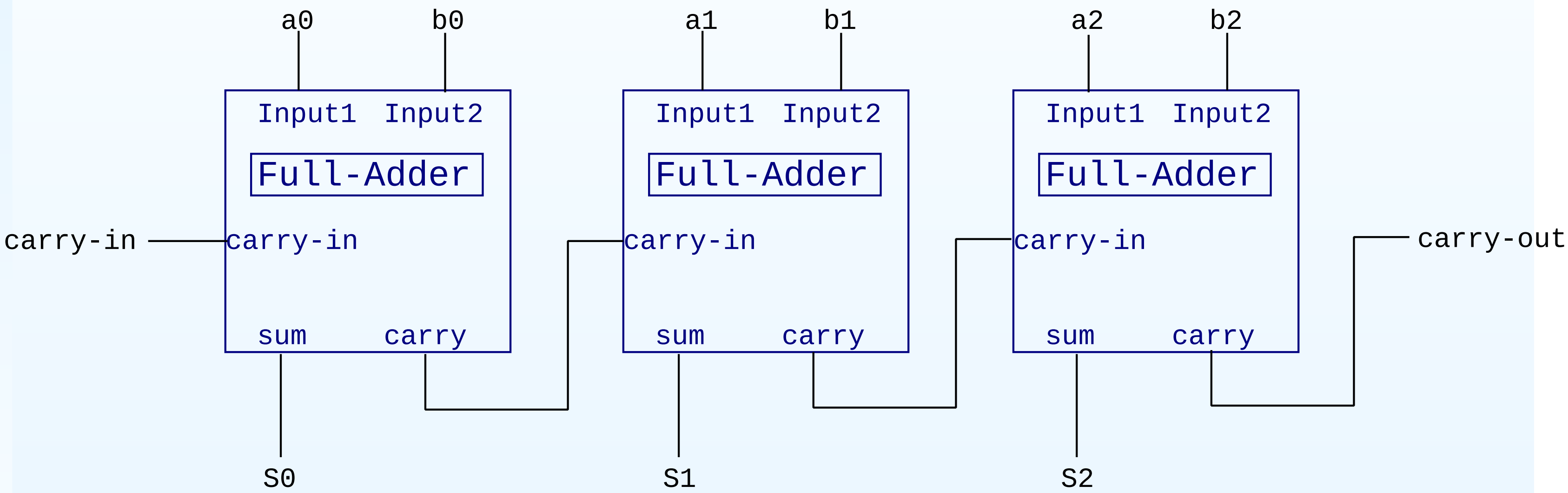
Computing Carries: Ripple Carry

- Hook up required number of full adders.
- As carry is calculated, passed to next bit.



Computing Carries: Carry Lookahead

- First calculate what carry bits would be, based on previous bits
- Another way to specify carry-out is: carry-out of any full-adder (C_n) is true if:
  - carry-out of the first half-adder ($A_n B_n$) is true, or
  - either one of the inputs and the carry-in is true: $(A_n + B_n)C_{n-1}$
- So: $C_n = A_n B_n + (A_n + B_n)C_{n-1}$ – a *recurrence relation*

Computer Science Foundations

Computing Carries: Carry Lookahead

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  - $C_2 = A_2 B_2 + (A_2 + B_2) C_1 \Rightarrow$
    $C_2 = A_2 B_2 + (A_2 + B_2)(A_1 B_1 + (A_1 + B_1) A_0 B_0)$
  - ...

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Computing Carries: Carry Lookahead

- Alternative could be done using $\oplus$:
  - For carry-out of the first full-adder (C_0), it's the carry-out of the full-adder's first half-adder OR'd with the carry-out of the second:
    $C_0 = C_{0_h} \vee C_{0_f} \Rightarrow$
    $C_0 = A_0 B_0 \vee S_0 C_{in} \Rightarrow$
    $C_0 = A_0 B_0 \vee (A_0 \oplus B_0) C_{in}$
  - C_1 is computable based on C_0 in the same way:
    $C_1 = A_1 B_1 \vee (A_1 \oplus B_1) C_0 \Rightarrow$
    $C_1 = A_1 B_1 \vee (A_1 \oplus B_1)(A_0 B_0 \vee (A_0 \oplus B_0) C_{in})$
  - Can generalize to n bits
- But better to keep with ANDs and ORs

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Trade-offs for Types of Carry Propagation

- Ripple carry has larger delay.
  - Ripple carry has delay as more significant binary digits wait for results from less significant digits.
  - Carry lookahead uses a sum of products to get result for each carry, so only a two gate delay (have an AND layer and an OR layer).
- Complexity of circuit.
  - Ripple carry requires only connecting carry-out to next carry-in.
  - Number of AND gates and number of inputs to OR gate is on the order of the number of digits for carry lookahead (i.e., $O(n)$, where n is the number of digits)

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Combining Ripple Carry and Carry Lookahead

- Minimize complexity of carry lookahead by only using it on a small number of bits in a group.

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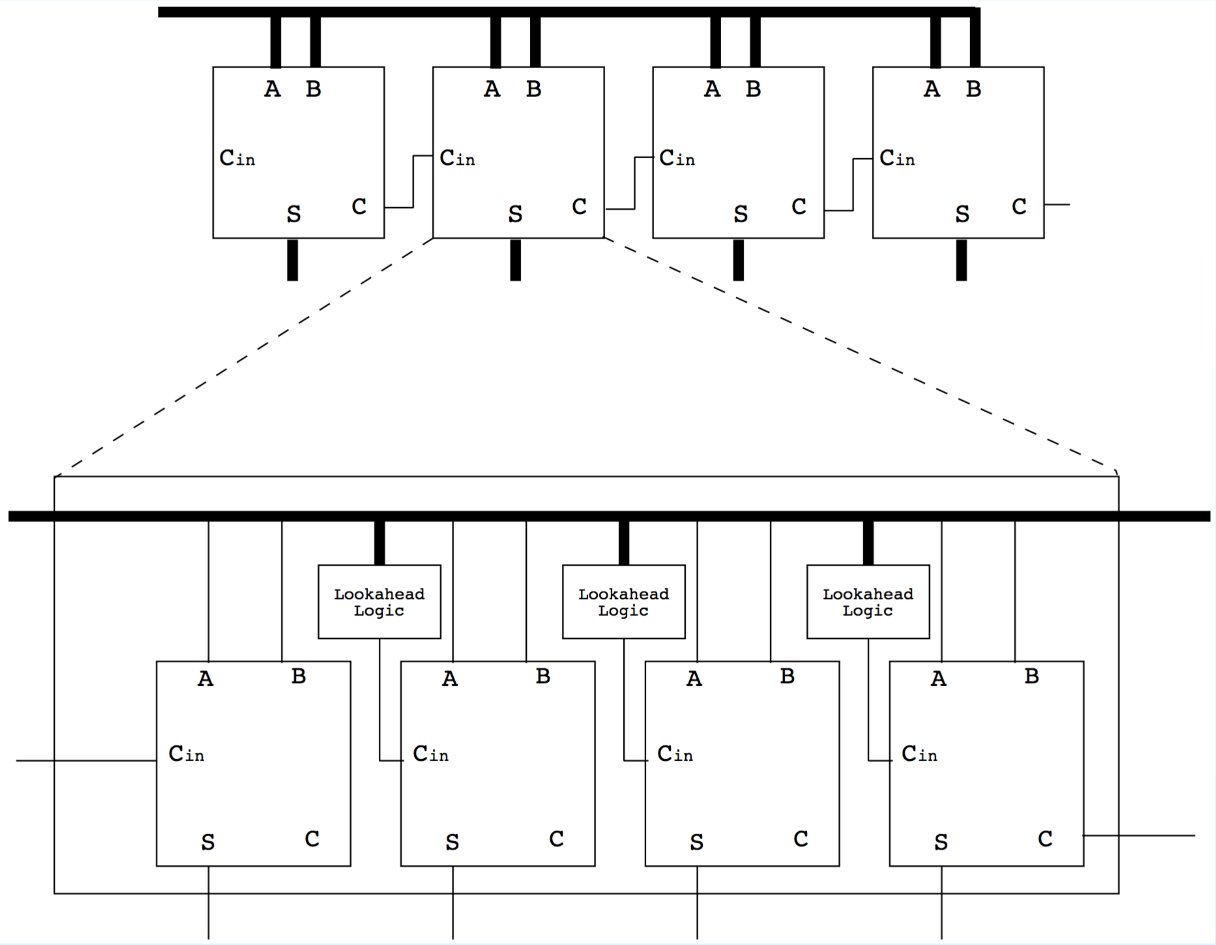

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- Sometimes what looks hard to implement (carry lookahead) may not be (2 layers of gates)

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